Gravitational Lensing

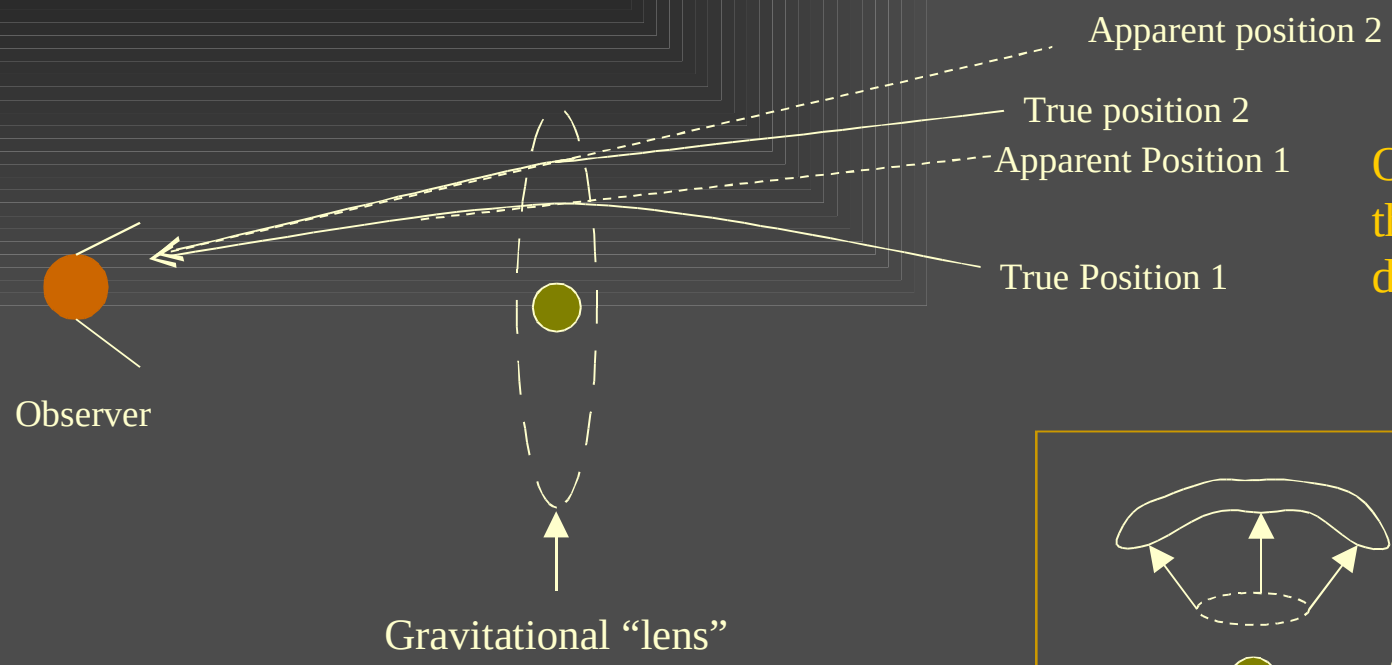
Probing Dark Matter, Dark Energy and Large-scale Structure

- Strong lensing:
  - Multiply imaged QSOs
  - Arcs: background galaxies lensed by clusters

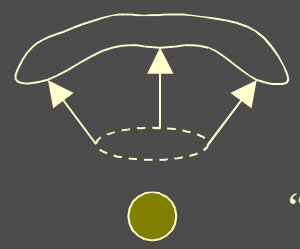
- Weak lensing:
  - Galaxy-Galaxy lensing: Probing Dark Matter Halos (and gal-QSO) and small-scale power
  - Cluster lensing: Mass Distribution on Mpc scales
  - Large-scale lensing: Power spectrum and Bias (cosmic shear)

Gravitational Lensing: what are the effects?

Basically, the same effects that occur in more familiar optical circumstances: magnification and distortion (shear)



Objects farther from the line of sight are distorted less.

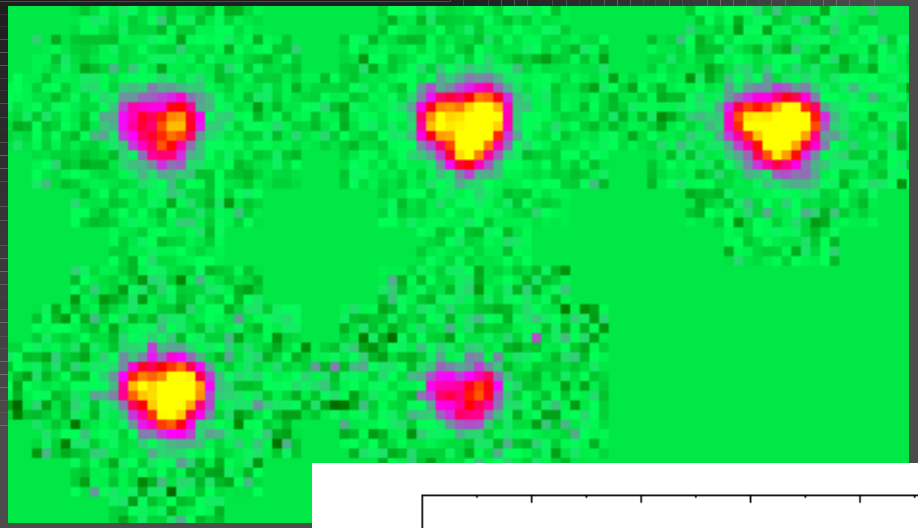


“Looking into” the lens: extended objects are tangentially distorted...

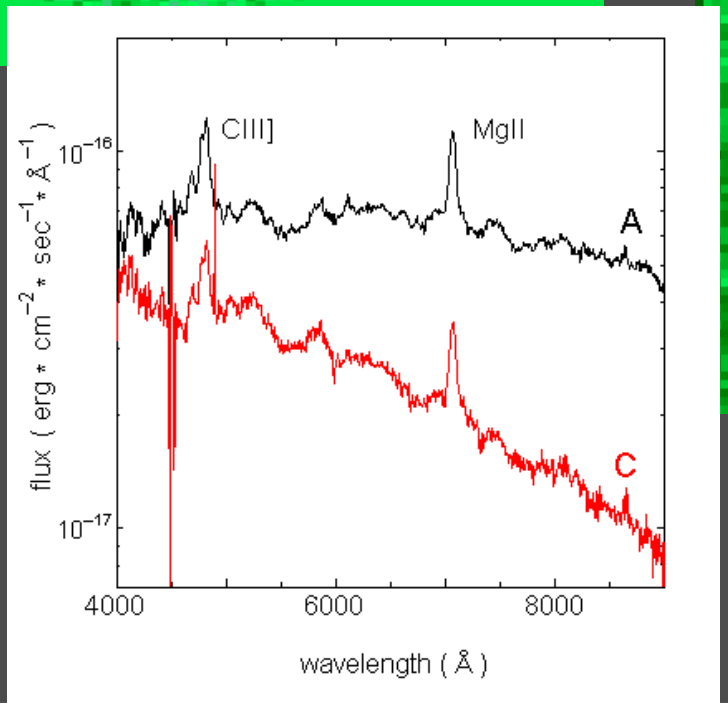
Lensing conserves surface brightness: bigger image magnified

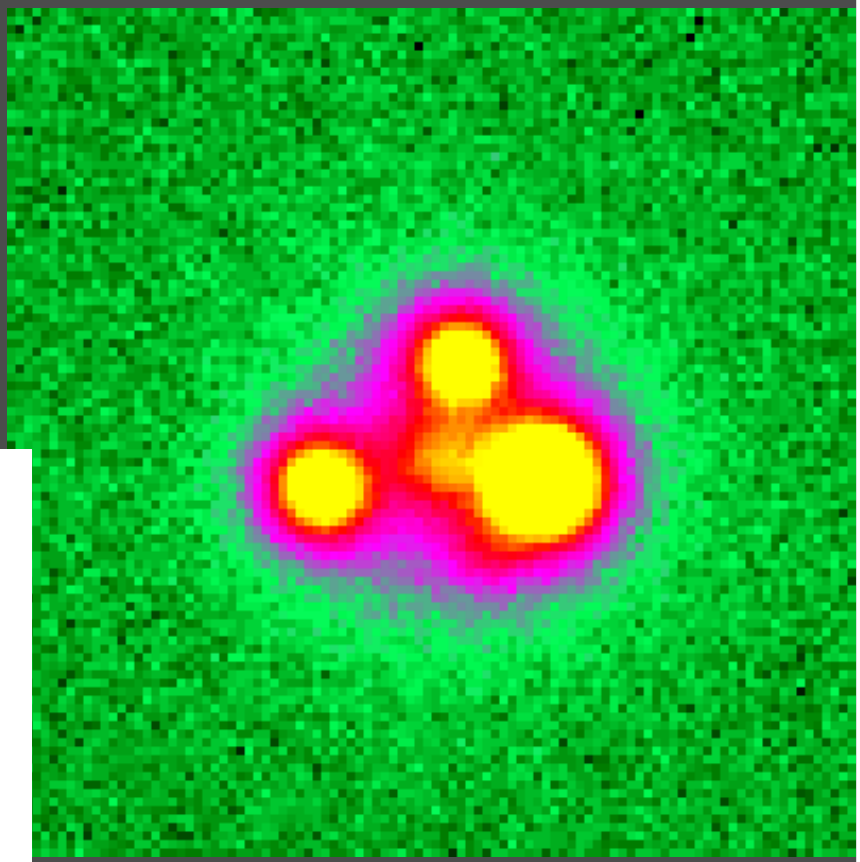
SDSS: Strongly Gravitationally Lensed QSO

SDSS image in 5 bands



Keck spectra





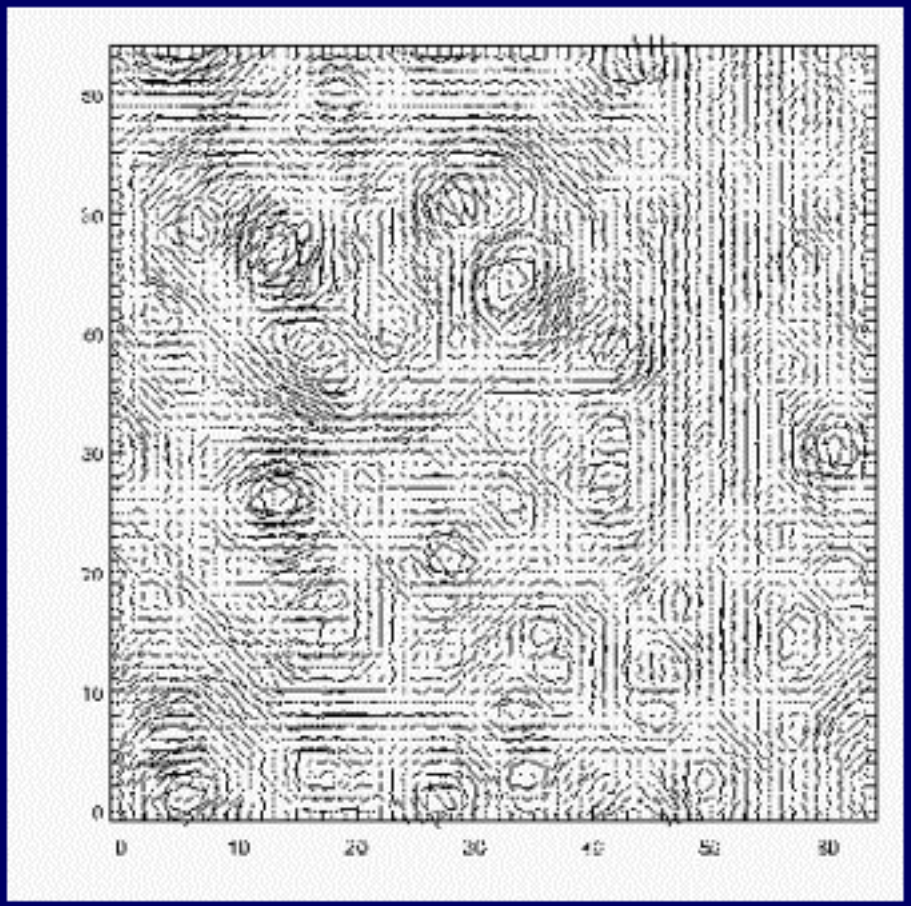
Magellan follow-up image: I band

Inada, etal

Giant arcs: distant galaxy images strongly distorted by foreground cluster

Scientific Promise of Weak Lensing

From the statistics of the shear field, weak lensing provides:



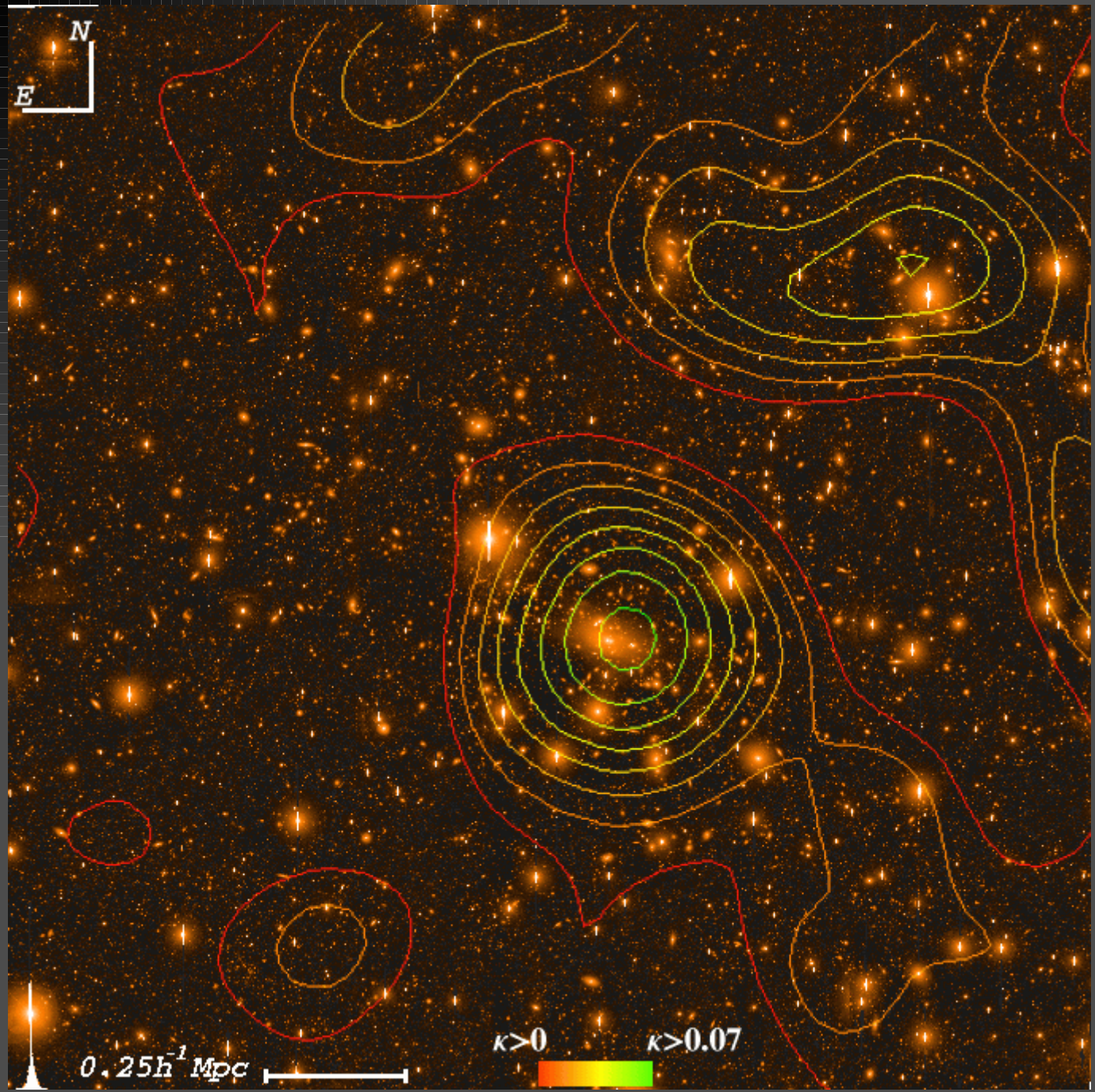
Jain et al. 1997, 1x1 deg

- Mapping of the distribution of Dark Matter on various scales
- Measurement of cosmological parameters, breaking degeneracies present in other methods (SNe, CMB)
- Measurement of the evolution of structures
- Test of gravitational instability paradigm
- Test of General Relativity in the weak field regime
- a mass-selected cluster catalog

Abell 3667

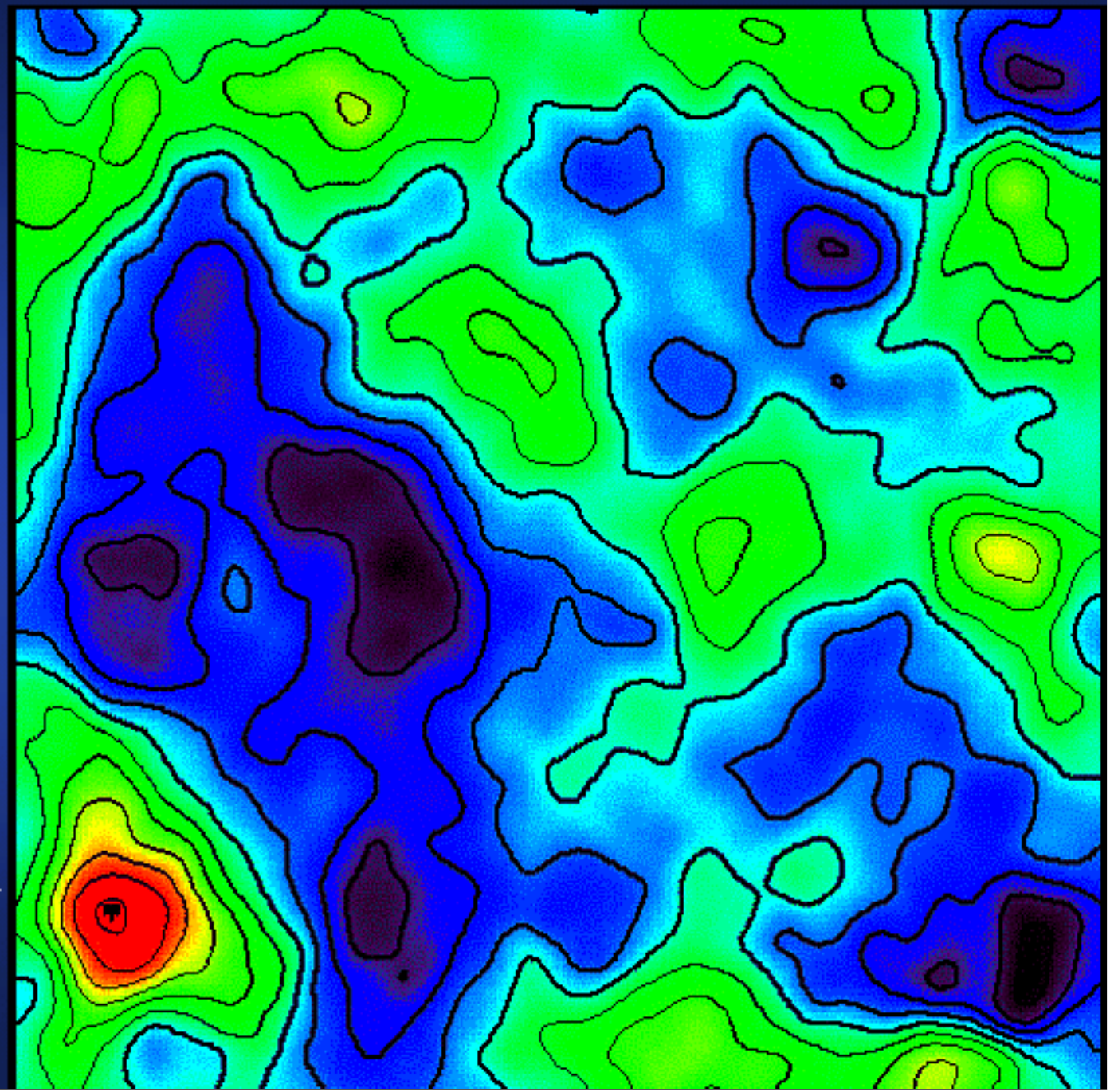
Weak Lensing Mass Map

z = 0.05

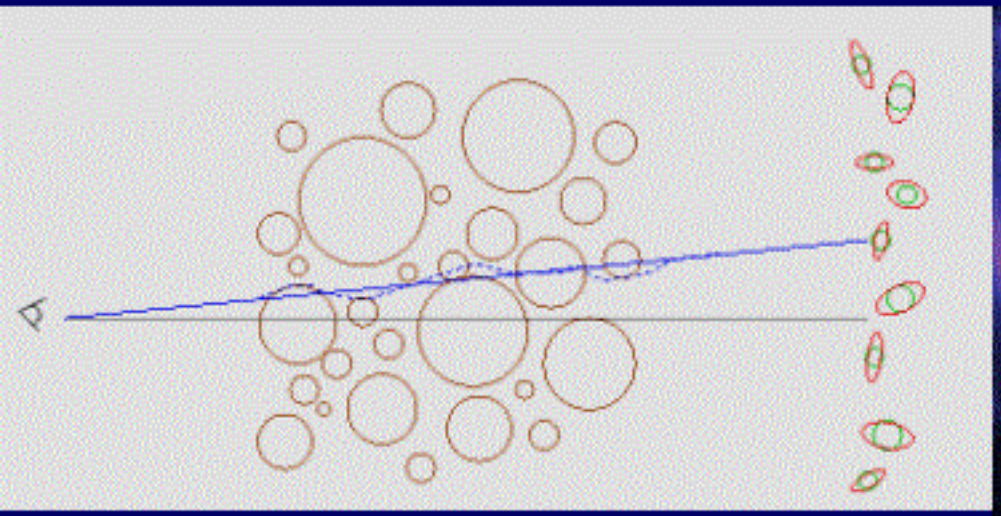


Joffre, etal

Cluster detected in weak lensing shear map



Weak Lensing by Large-Scale Structure



Distortion Matrix:

$$\Psi_{ij} = \frac{\partial \delta\theta_i}{\partial \theta_j} = \int dz\, g(z) \frac{\partial^2 \Phi}{\partial \theta_i \partial \theta_j}$$

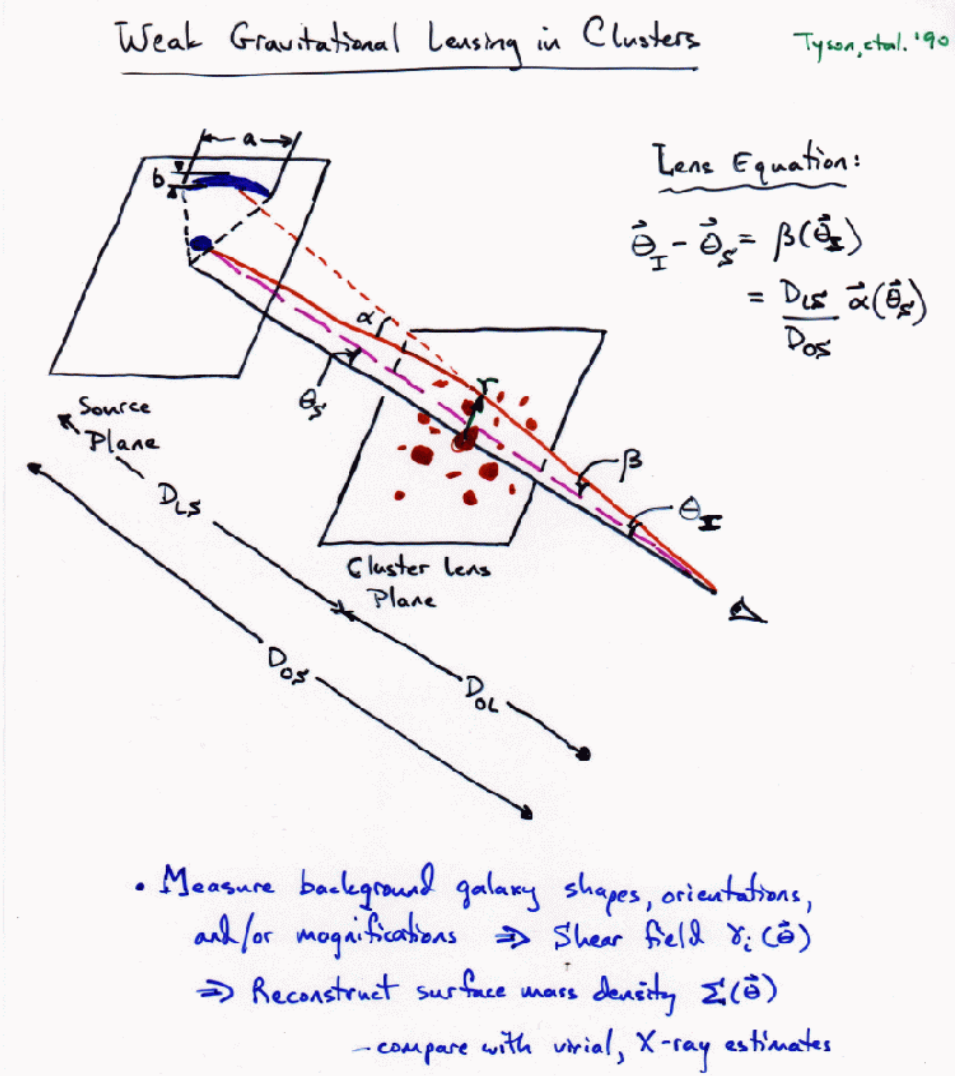
Theory

→ Direct measure of the distribution of mass in the universe, as opposed to the distribution of light, as in other methods (eg. Galaxy surveys)

2 transparencies on lens equation and critical density

Weak Gravitational Lensing

Amplification, Convergence, and Shear



Bend angle: $\beta_i = \partial_i \Psi$

where $\nabla^2\Psi = 2\Sigma/\Sigma_c = 2\kappa$ **(2D Poisson eqn.)**

Amplification Matrix:

$$M_{ij} = \partial\theta^S_i/\partial\theta^I_j = \delta_{ij} - \partial\beta_i/\partial\theta_j = \delta_{ij} - \partial^2\Psi/\partial\theta_i\partial\theta_j$$

$$= \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix}$$

where $\gamma_1 = (\partial_1^2\Psi - \partial_2^2\Psi)/2 \qquad \gamma_2 = \partial_1\partial_2\Psi$

Amplification:

$$A = (\det M)^{-1} = (1-\kappa-\gamma)^{-1}(1-\kappa+\gamma)^{-1}$$

where

$$\gamma = (\gamma_1^2+\gamma_2^2)^{1/2}$$

$$\kappa(\theta) = \int d^2\theta' \, \mathrm{Re}[D(\theta - \theta')\,(\gamma_1 + i\gamma_2)] \quad \text{inversion}$$

Transparency on image lensing

Note: $x = \theta$

Procedure

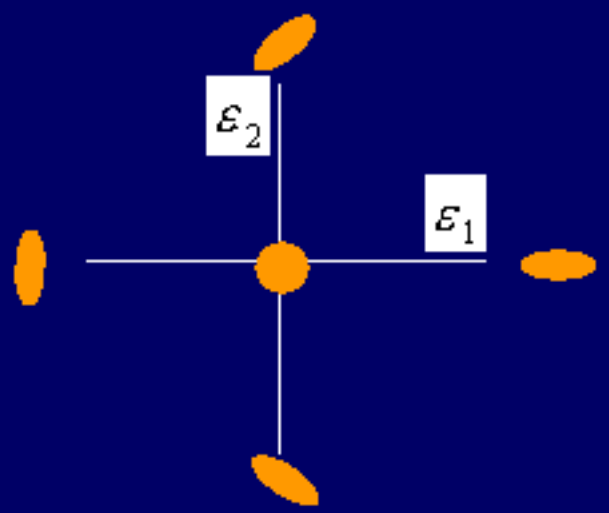
Shear from Galaxy shapes: $I'(x_i)=I(M_{ij}x_j)$

Quadrupole moments:

$$Q_{ij} = \int d^2x\, x_i x_j\, w(\vec{x}) I(\vec{x})$$

Ellipticity:

$$\varepsilon_1 = \frac{Q_{11} - Q_{22}}{Q_{11} + Q_{22}}, \quad \varepsilon_2 = \frac{2Q_{12}}{Q_{11} + Q_{22}}$$



Shear:

$$\Psi_{ij} = \begin{pmatrix} 1 - \kappa - \gamma_1 & \gamma_2 \\ \gamma_2 & 1 - \kappa + \gamma_1 \end{pmatrix}$$

Relation:

$$\langle \varepsilon_i \rangle = P^{\gamma} \gamma_i$$

Unweighted moments: $P^{\gamma} = 2(1 - \langle e^2 \rangle/2)$ shear polarizability

The names of the game:

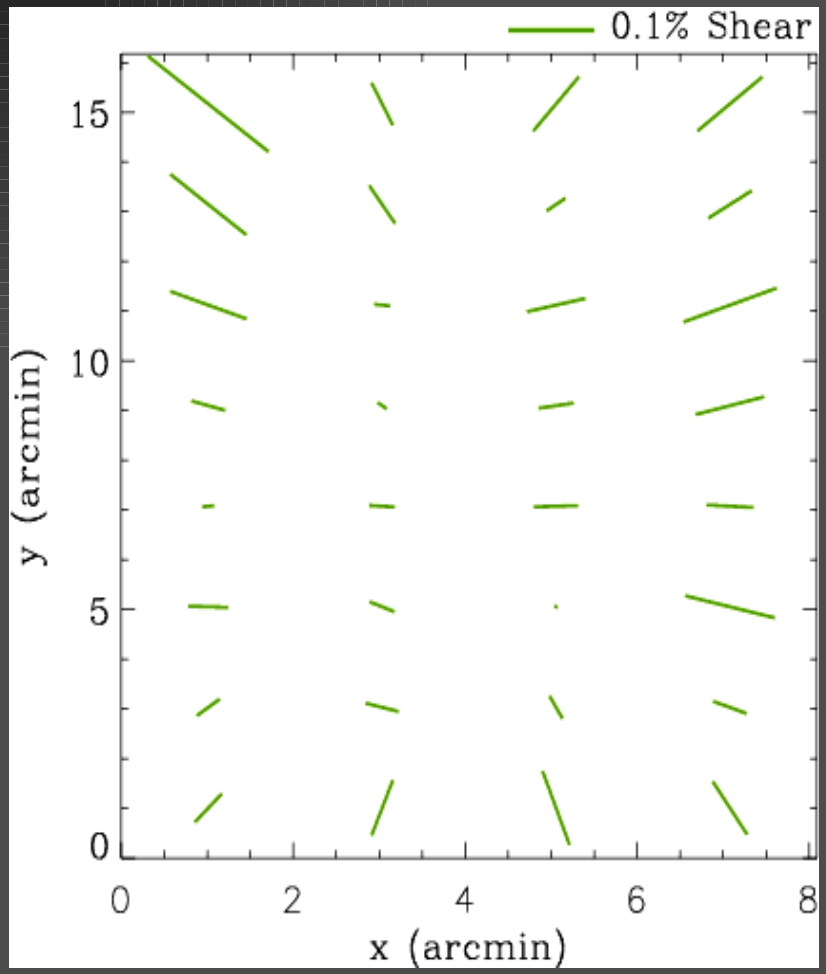
3. Going deep enough to beat down shape noise: $\langle e^2 \rangle / N^{1/2}$

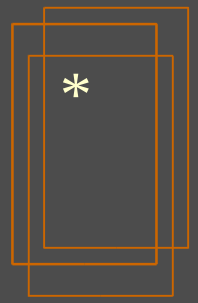
2. Correcting for large systematic effects: PSF anisotropy, using star images

3. Choice of weight function (adaptive Gaussian)

Cosmic shear signal is small: controlling systematics is critical

Instrumental Distortion:
Correct using Astrometric standard stars→pixel transform

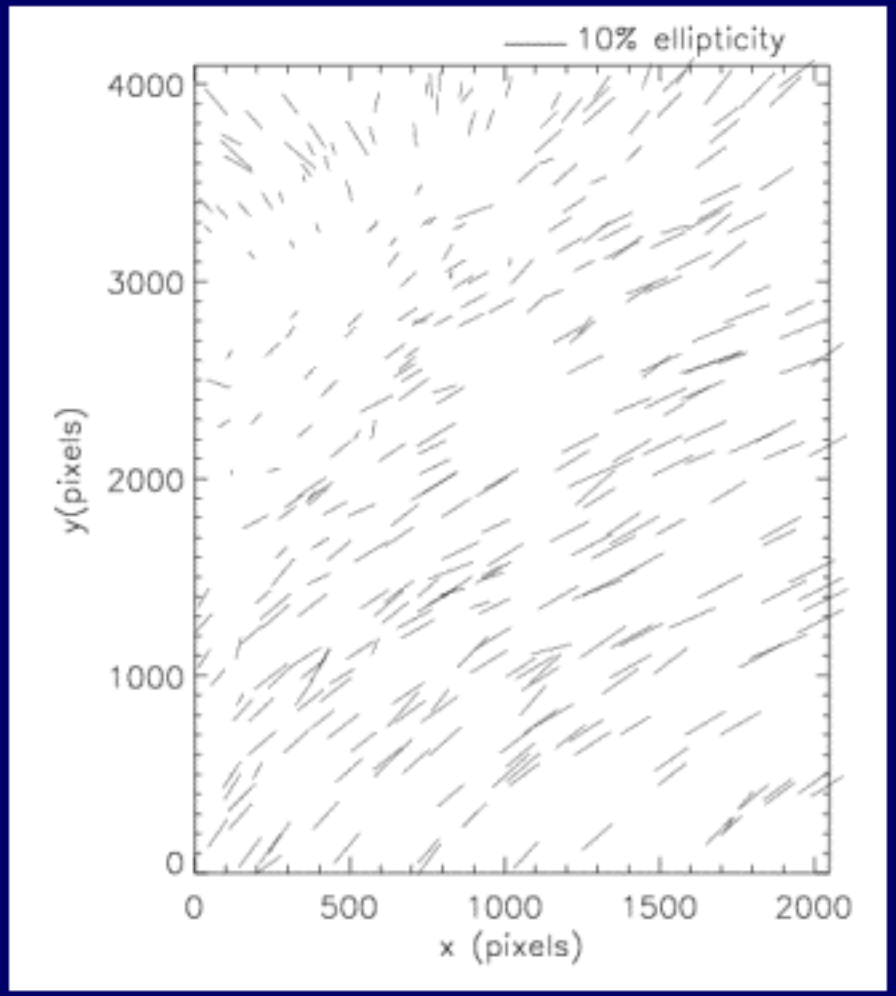


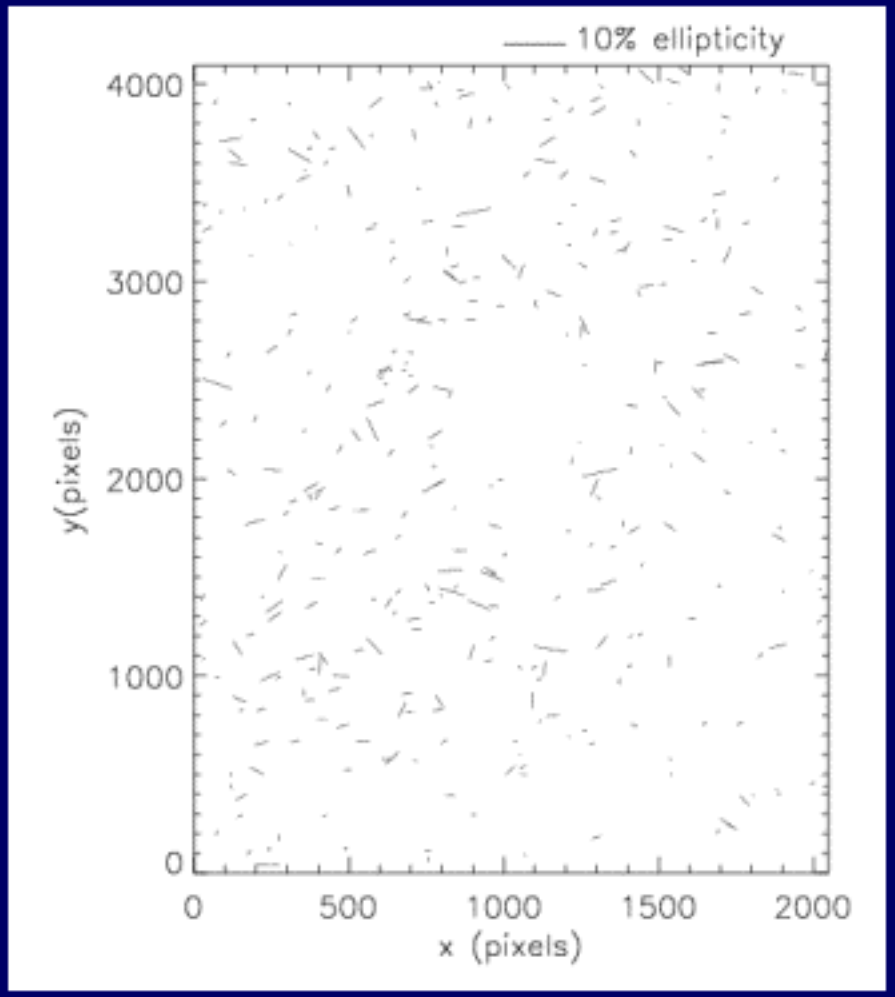


Dithered fields

Transparencies: PSF, mag-size

PSF anisotropy





3-10% rms reduced to ≈0.1%

PSF Correction Methods

KSB Method: (Kaiser, Squires & Broadhurst 1995)

PSF Anisotropy:

$$\varepsilon_g = \varepsilon'_g - \frac{P_g^{sm}}{P_¿^{sm}} \varepsilon_¿$$

PSF Smear & Shear Calibration:

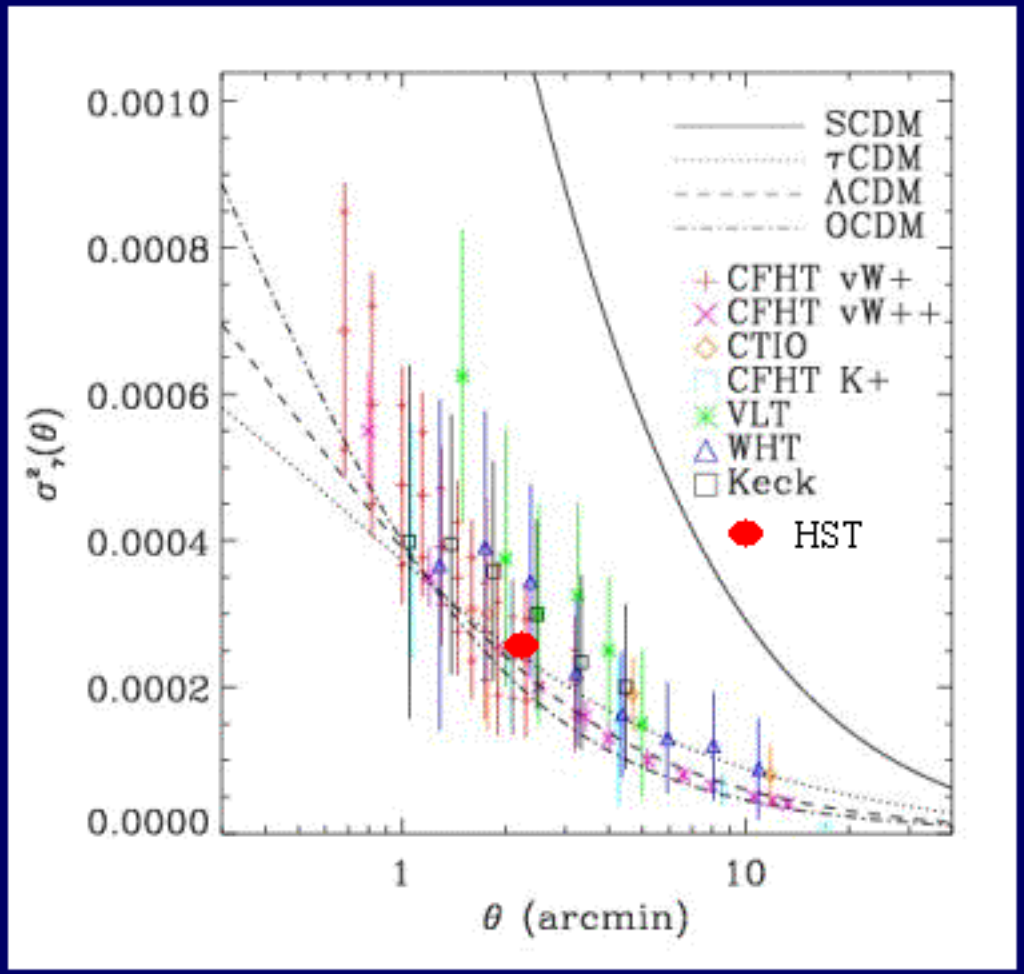
$$\gamma = \left(P^{\gamma}\right)^{-1} \varepsilon_g$$

Other Methods: Kuijken (1999), Kaiser (1999), Rhodes, Refregier & Groth (2000), Refregier & Bacon (2001), Bernstein & Jarvis (2001)

or can instead apply an image transform which recircularizes the PSF

See Bernstein: now use shapelets to deconvolve PSF

Current Observational Status



Shear variance in circular cells:

$\sigma^2_\gamma(\theta)=\langle\gamma^2\rangle$

→ Different measurements are consistent

→ In agreement with cluster-normalised CDM model

→ measure of the amplitude of mass fluctuations:

$\sigma_8(\Omega_m/0.3)^{0.5}=1.07 \pm 0.23$

Cluster counts (Viana & Liddle, Eke et al.): $\sigma_8(\Omega_m/0.3)^{0.5}=1.02\pm 0.11$

→ In agreement, test of primordial non-gaussianity

LS Shear estimators:

Variance of shear

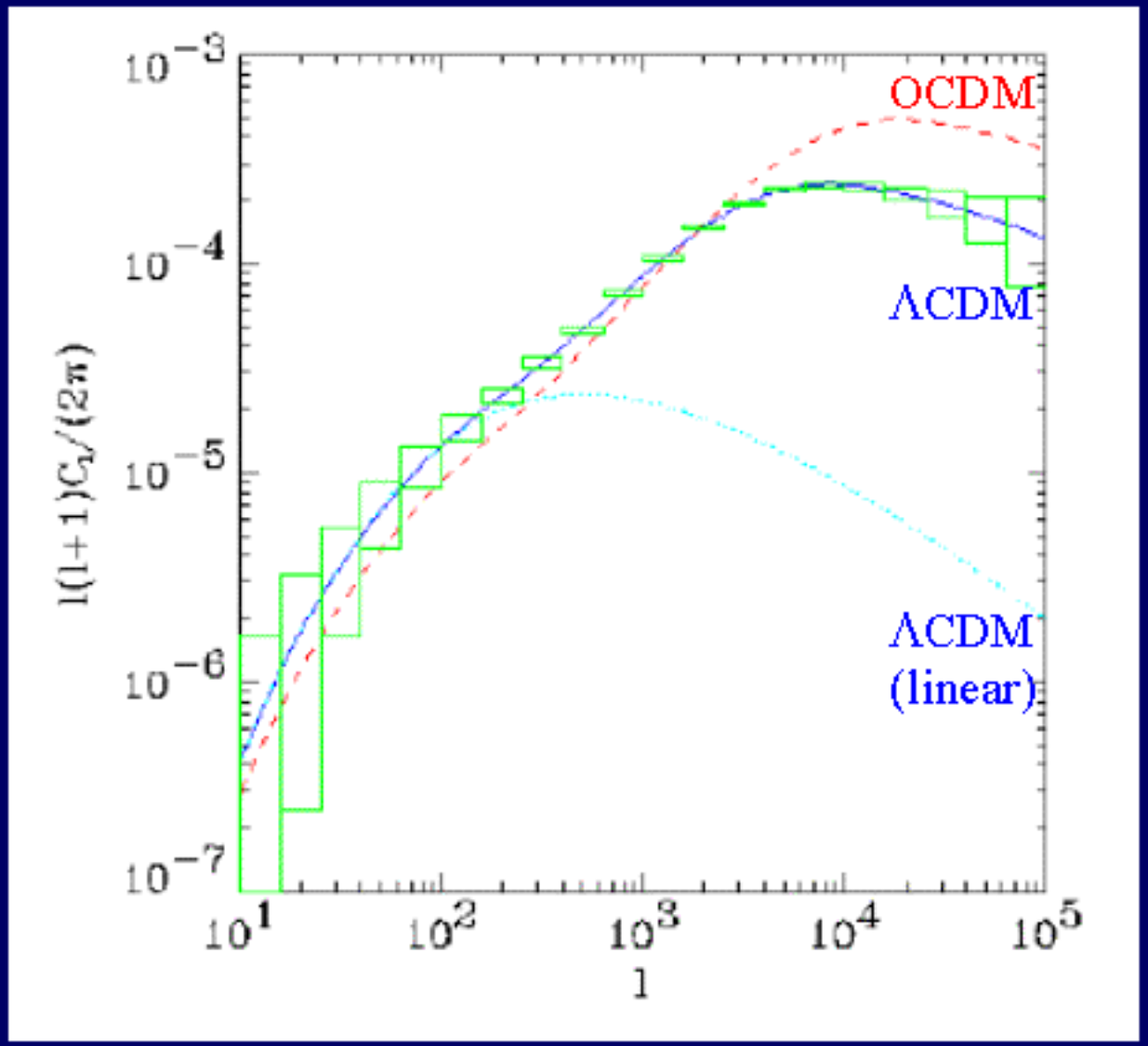
Shear-shear corr. Fn.

Map statistic

Convergence power spectrum (linear regime)

Hu formula for P(k) errors

Weak Lensing Power Spectrum

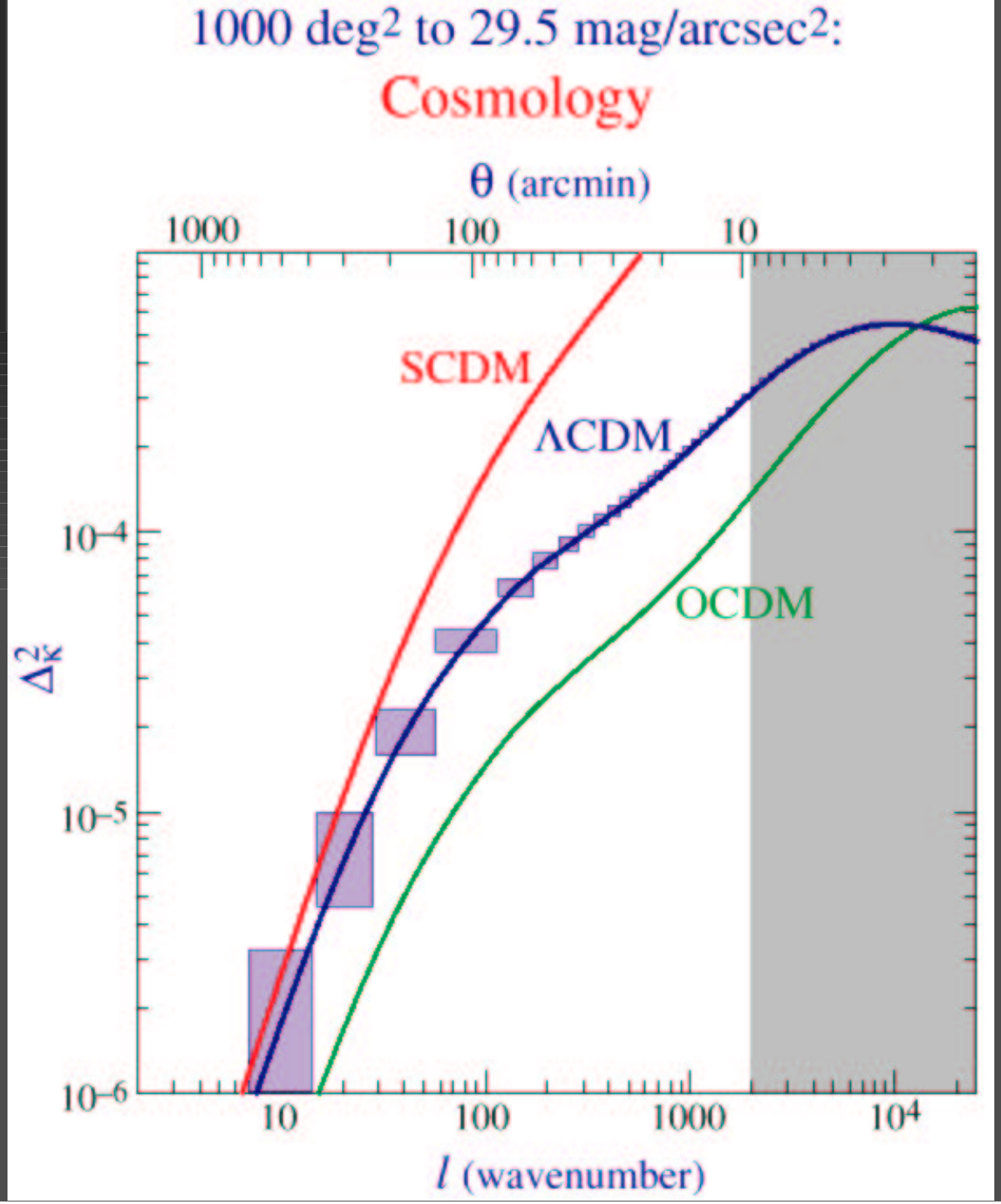


Future surveys:
Megacam, Subaru, VISTA, LSST, WHFRI, SNAP, etc

→ Measure cosmological parameters (σ_8, Ω_m, Ω_Λ, Γ, etc)
→ very sensitive to non-linear evolution of structures

SNAP WF survey [300 deg^2 ; 100 g arcmin^{-2}; HST image quality]

Weak Lensing Map of Mass Power Spectrum



Hu

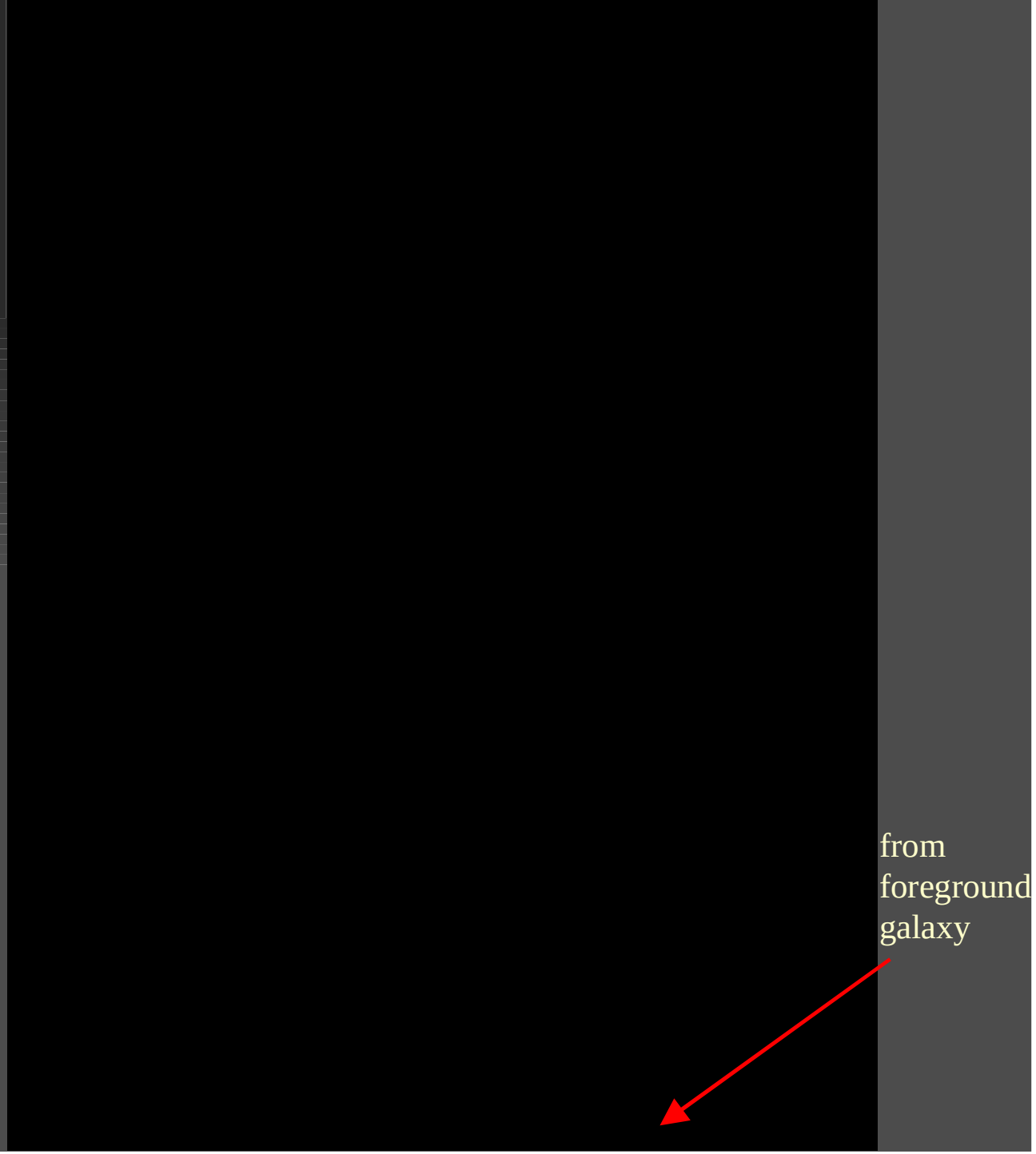
Galaxy-Galaxy Lensing in early SDSS Data

Galaxy-mass Correlation function

~31,000 foreground galaxies with measured redshifts

~10^6 background galaxy shapes (18<r'<22)

Fischer, etal
McKay, etal



Measurement of the Galaxy-Mass correlation function: the GMCF

$\xi_{\text{Galaxy-Galaxy}}$: directly measured by SDSS LSS

$\xi_{\text{Mass-Mass}}$: directly predictable by N-body simulations

$\xi_{\text{Galaxy-Mass}}$: now measured by SDSS lensing

Encoded in their relationship is the 'bias' between light and mass

$$w_{g\gamma}(\theta) = 12\Omega_m b_g \int dk\, (k/2\pi) P_\delta(k) \int dw (1-w)^2\, G_b(w) W_f(w) J_0(wk\theta)$$

Infer $\Omega_m b_g \sigma_8^2$ 1/4 to 1/3 from shear and $b_g \sigma_8$ 1 from foreground clustering

Consistent with low-density universe and modest bias

Physical Effects of Dark Energy

Dark Energy affects expansion rate of the Universe:

$$H^2 = \frac{8\pi G}{3}(\rho_M + \rho_X)$$

$$H(z)^2 = H_0^2 \left[\Omega_M (1+z)^3 + \Omega_X \exp\left[3\int_0^z (1+w(x))d\ln(1+x)\right]\right]$$

Comoving Distance: $r(z) = \int dx/H(x)$

In a flat Universe:

Luminosity Distance: $d_L(z) = r(z)(1+z)$

Angular diameter Distance: $d_A(z) = r(z)/(1+z)$

Comoving Volume Element: $dV/dzd\Omega = r^2(z)/H(z)$

Physical Observables: probing DE

1. Luminosity distance vs. redshift: $d_L(z)$ $m(z)$

   Standard candles: SNe Ia Hubble diagram

2. Angular diameter distance vs. z: $d_A(z)$

   Alcock-Paczynski test: Ly-alpha forest; redshift correlations

3. Number counts vs. redshift: $N(M,z)$

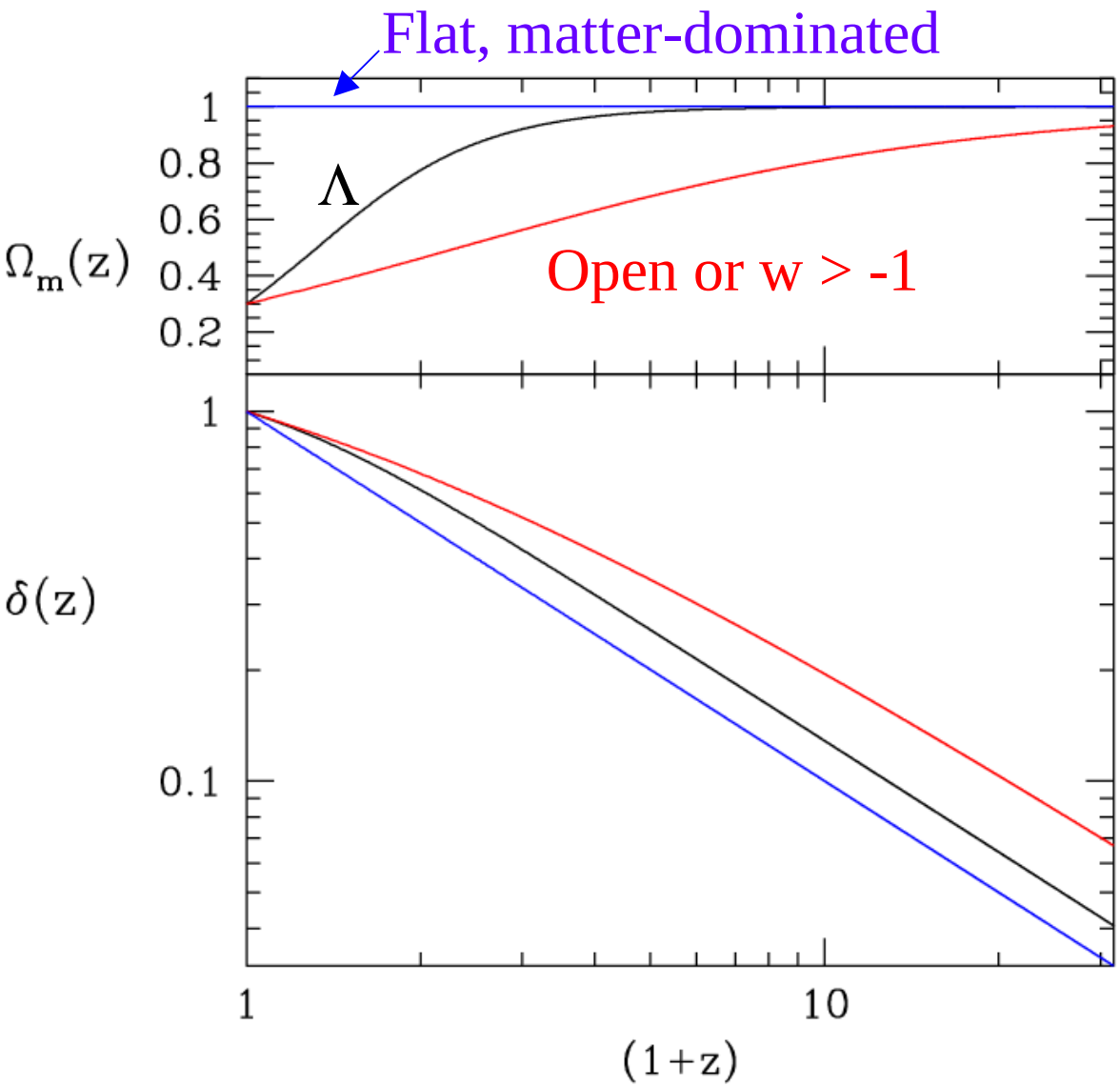
   probes:

   *Comoving Volume element $dV/dzd\Omega$

   *Growth rate of density perturbations $\delta(z)$

   Counts of galaxy halos and of clusters; QSO lensing

Growth of Density Perturbations



Holder

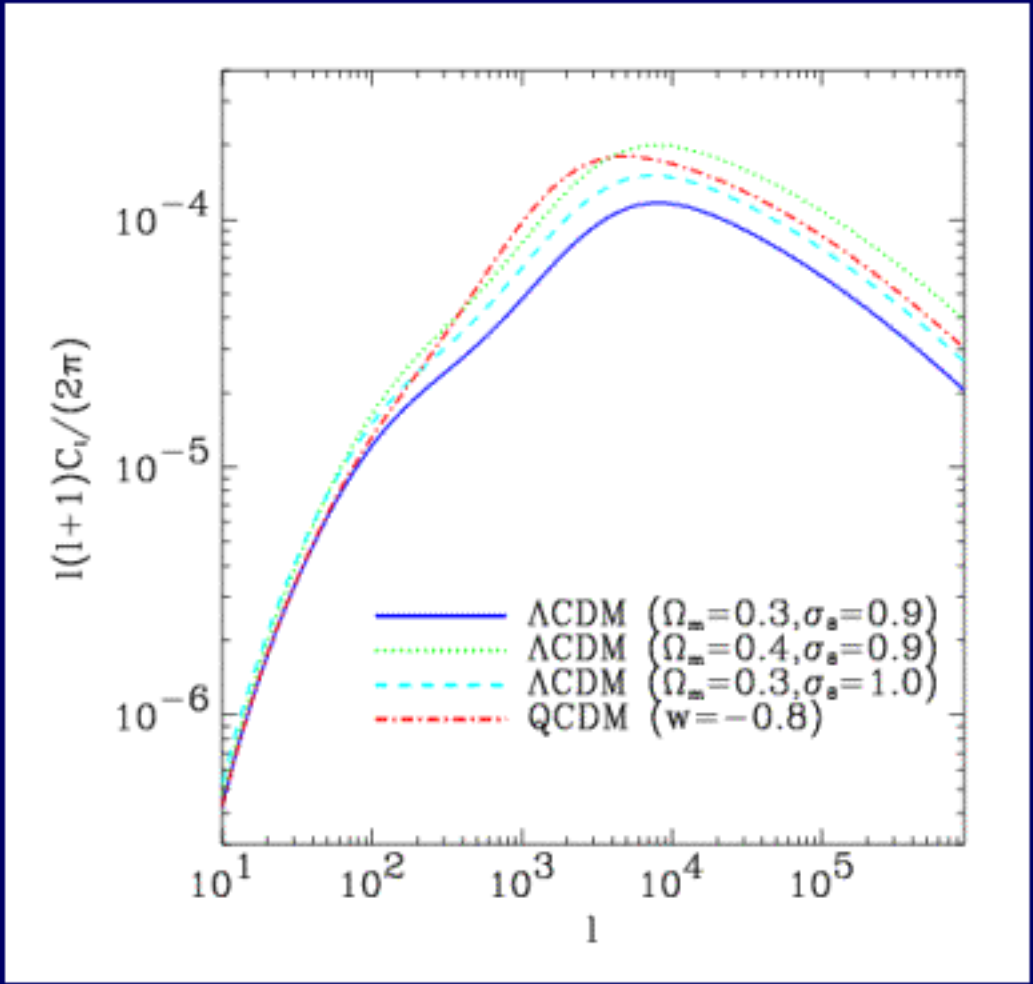
Dark Energy

Effect of Dark Energy on Weak Lensing Statistics:

- Modifies the Angular-Diameter Distance +
- Modifies the rate of growth of structures +
- Modifes the shape of the linear matter power spectrum -

Cf. Benabed & Bernardeau 2001
Huteterer 2001
Refregier et al. 2001 (in preparation)

Power Spectrum with Dark Energy

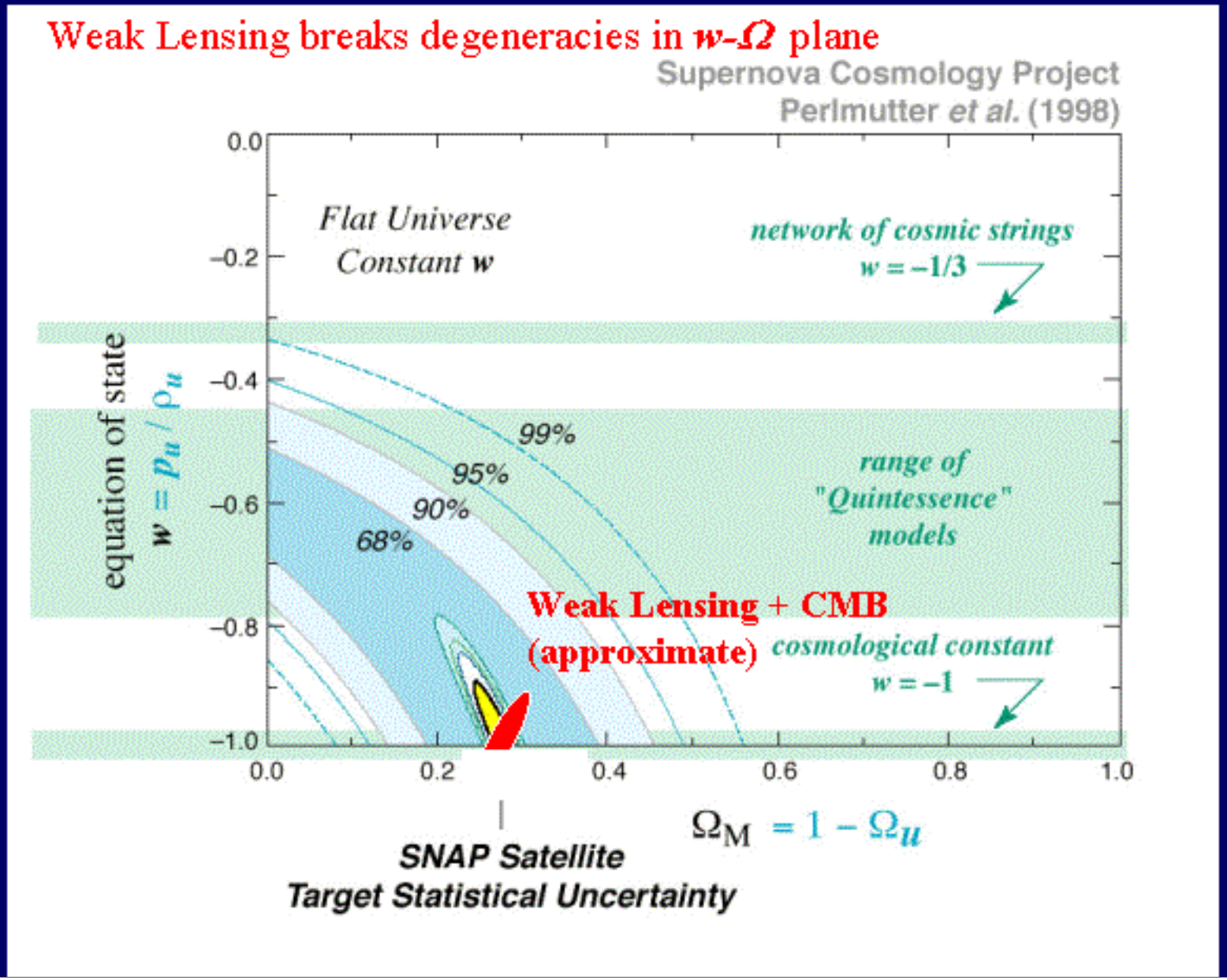


Use the non-linear power spectrum for quintessence models of Ma, Caldwell, Bode & Wang (1999)

→ The Dark Energy equation of state ($w=p/\rho$) can be measured from the lensing power spectrum

→ But, there is some degeneracy between w, Ω_M and σ_8

Complementarity of Weak Lensing and Supernovae



Good News and Bad News

Caveats:

- Very sensitive to Non-linear Power spectrum: need very accurate fitting formulae from N-body simulations
- Requires knowledge of the redshift distribution of the galaxies
- requires tight control of systematic effects

Additional information:

- Power spectrum for different redshift bins (tomography)
- High-order moments (skweness or bispectrum, etc)
- Mass-selected cluster catalogues

Large Synoptic Survey Telescope

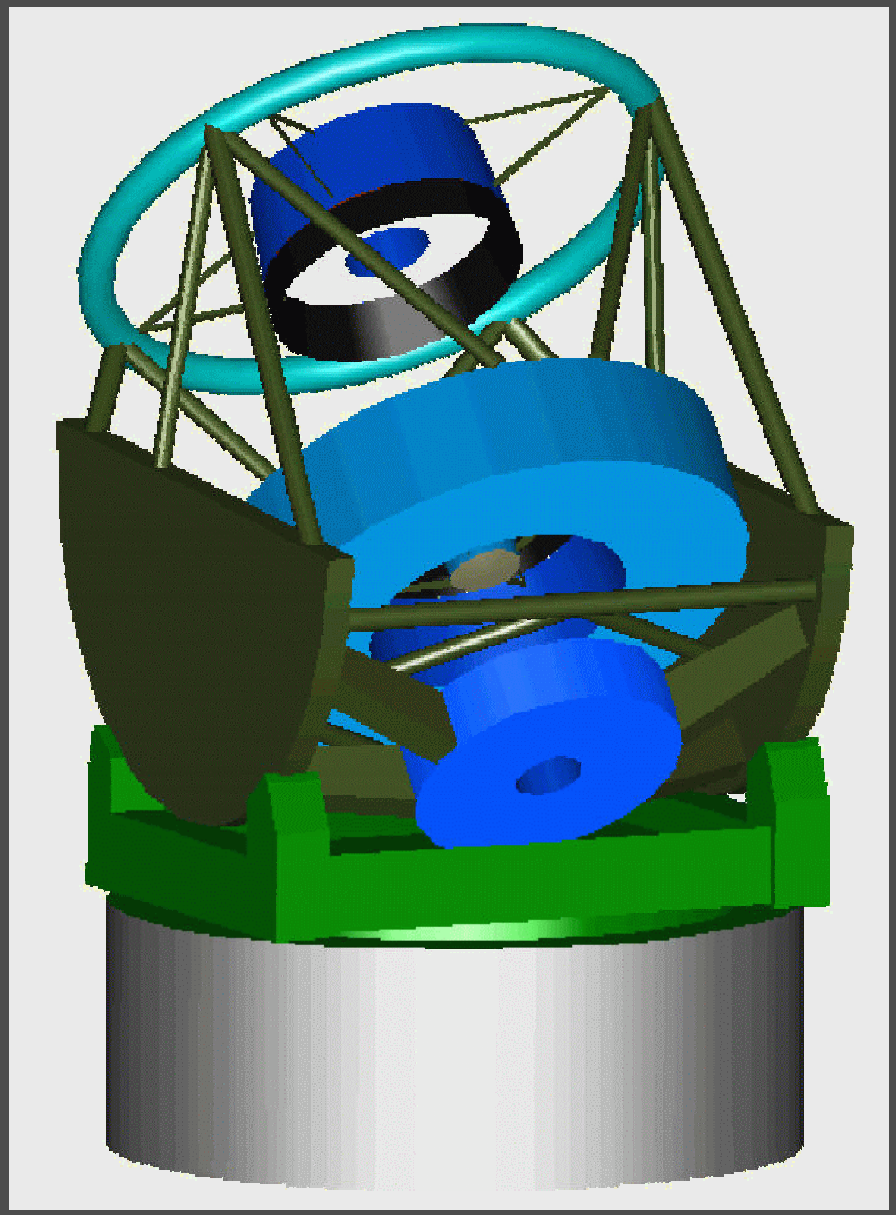
Proposed 8.5m ground based telescope with 7 sq. degree field

5000 Gbytes/night

Real-time analysis

“Celestial Cinematography”

Also: VISTA,
PANSTARRS,…



Billion-Pixel Observatories: Ground vs space

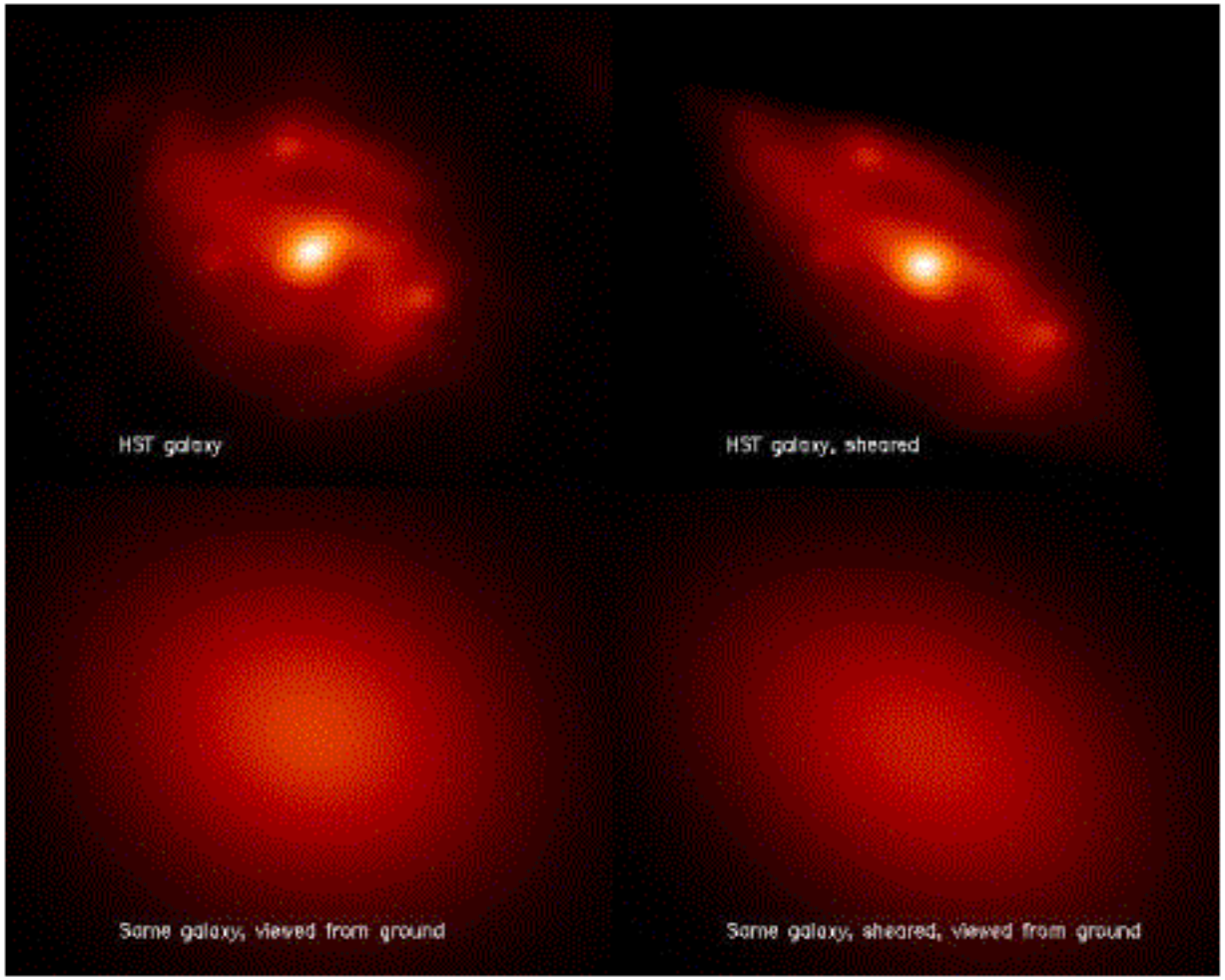
Large Synoptic Survey Telescope (LSST) and Supernova Acceleration Probe (SNAP)

	LSST	*SNAP*
Primary Mirror Diameter	8 meters (6.5-meter equivalent)	2.0 meters
CCD Field of View	7 square degrees	1 square degree
Number of CCD pixels	1.3×10^9	1.3×10^9
Angular Resolution	0.5”-0.7” (atmospheric seeing)	0.05”-0.1” (diffraction-limited)
Platform	Ground	Space
Nominal Exposure	20 sec.	300 sec.

Survey	Observing Dates	Sky Coverage (deg^2)	Resolved Galaxies per $arcmin^2$	Filter Bands, Nominal Depth	Total Galaxy Shapes ($x10^6$)	Median Seeing (arcsec FWHM)	Publications/Comments
CTIO Survey	1996-2000	75	4	R<23	1.0	1.0	Fully reduced, Jarvis et al., to be submitted late 2001
Sloan Digital Sky Survey (SDSS)	1999-2004	10,000	1.8	ugriz, r<22	70	1.2	McKay et al. 2001 (3% of data); too shallow for cosmic shear.
SDSS Southern Cap	2001-2004	300	5-10	ugriz, r<23.5?	5-10	1.2	
Red Cluster Survey (RCS)	1999-2002	100	10	Rz, R<24	4	0.75	Hoekstra, *et al.* (2002) is best 25% of R data.
VIRMOS/CFHT	1999-2002	16	18	UBVRI, I_{AB}<24.5	1.0	0.8	van Waerbeke et al. (2001) is 25% of I data.
Hawaii/CFHT	1998-2002?	9	30	VI, I<24	0.7	0.8	Kaiser, Wilson, & Luppino (2000) is 20% of data
Deep Lens Survey (DLS)	1999-2003	28	~20	BVRz, R<25?	2	0.7-0.9	50% of data taken.
CFHT Legacy Wide Synoptic	2002-2007	208	20-30	ugriz, I_{AB}<24.3	15-20	0.7?	Approval is imminent.
VISTA	2005-??	1000?	~20	I<24	70-100	0.6?	Dedicated 4-meter; funded, survey parameters not set.
Large Synoptic Survey Tel. (LSST)	2008?	10,000	~30	UBVRIZ, R<25?	1000	0.6?	Not yet funded or fully defined
Supernova Acceleration Probe (SNAP)	2008?	25	>500	B thru H, I<30	45	0.1	Not yet funded.
SNAP/Wide	2010?	1000	~200	I<28	700	0.1	Possible lensing survey mode.

Table 1: Present and future largest weak-lensing surveys.

Advantage of Space for Weak Lensing

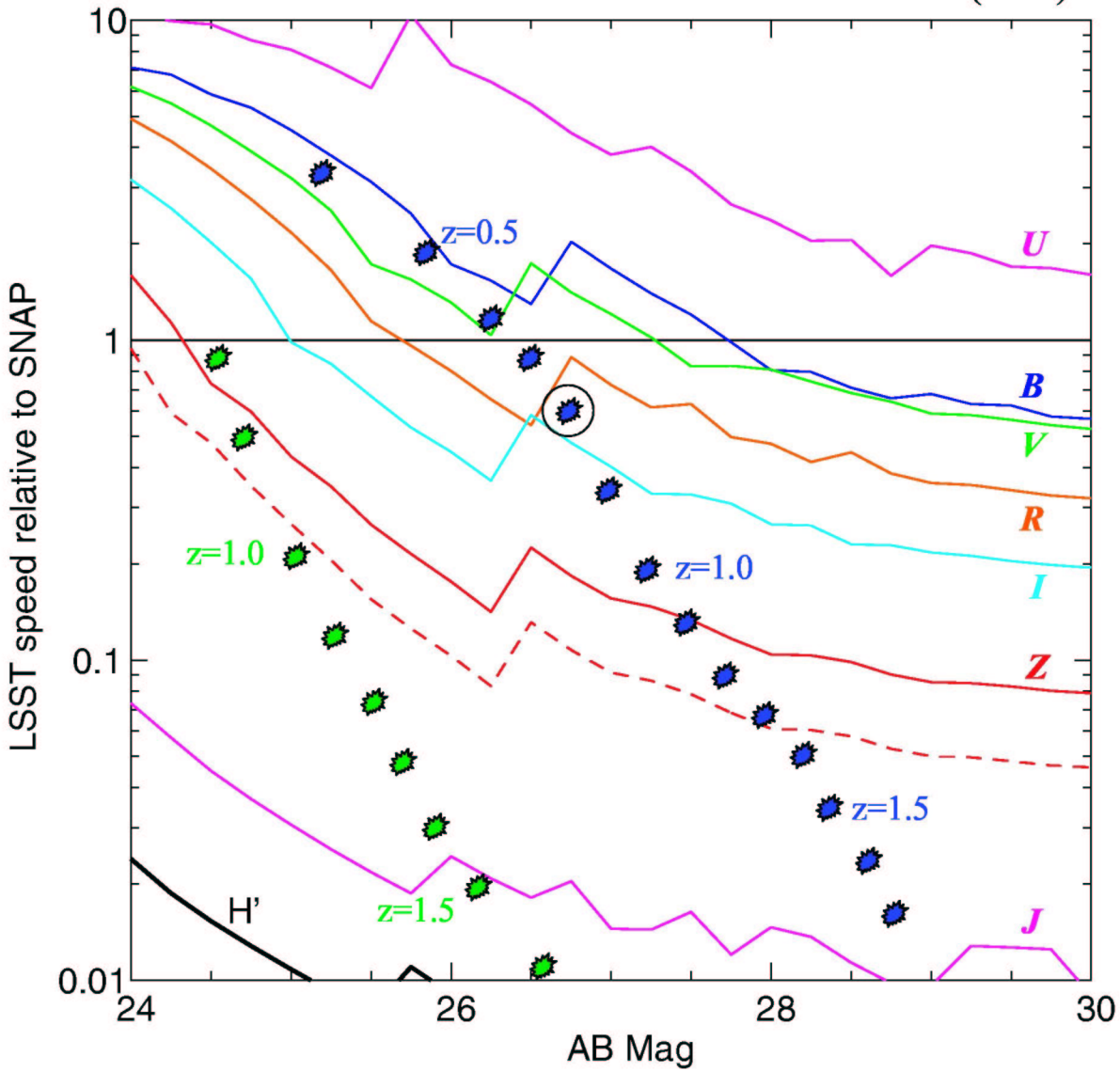


- larger surface density of resolved galaxies
- very reduced impact of the PSF smearing
- more shape information

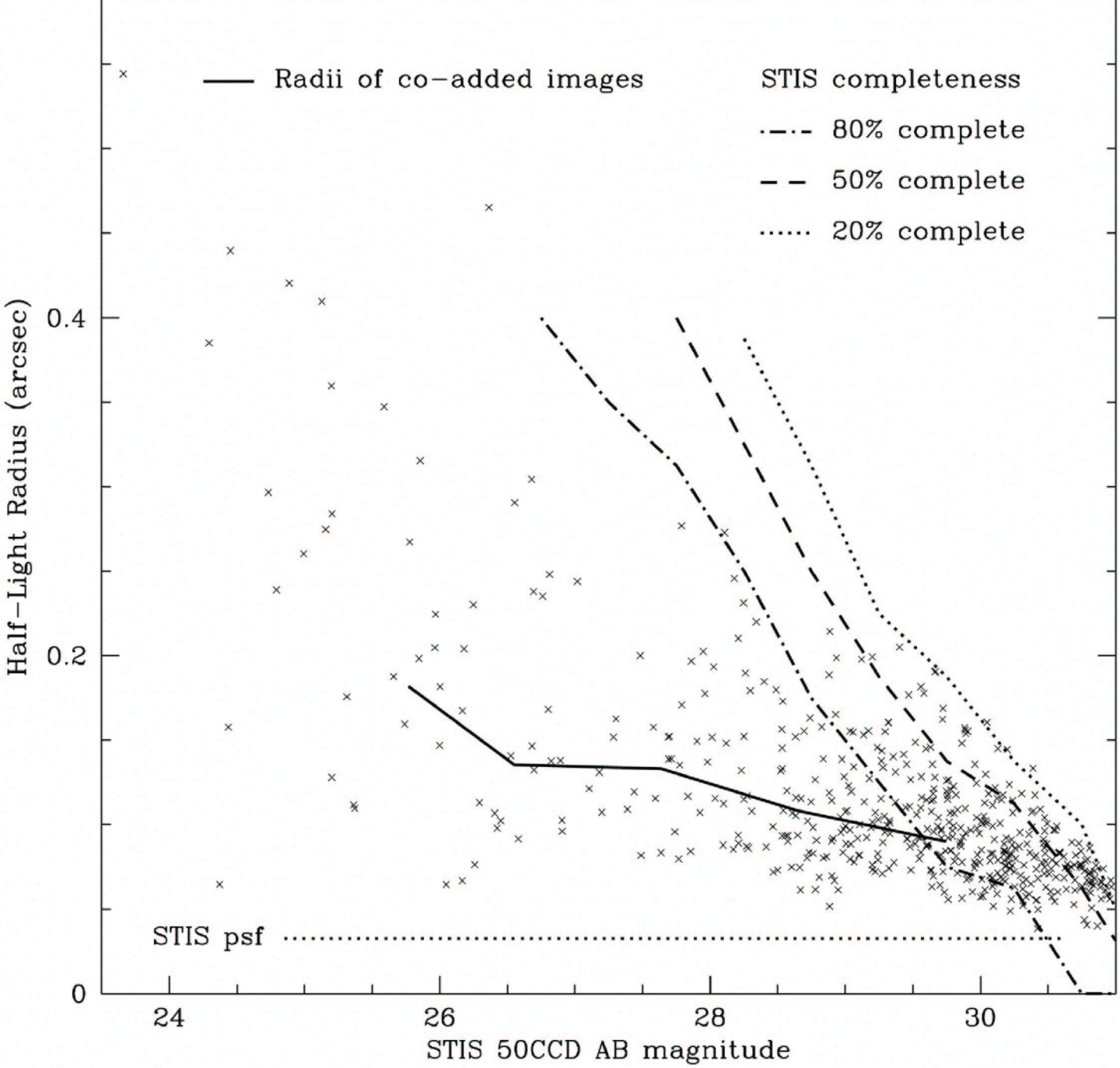
Strengths of SNAP for Weak Lensing

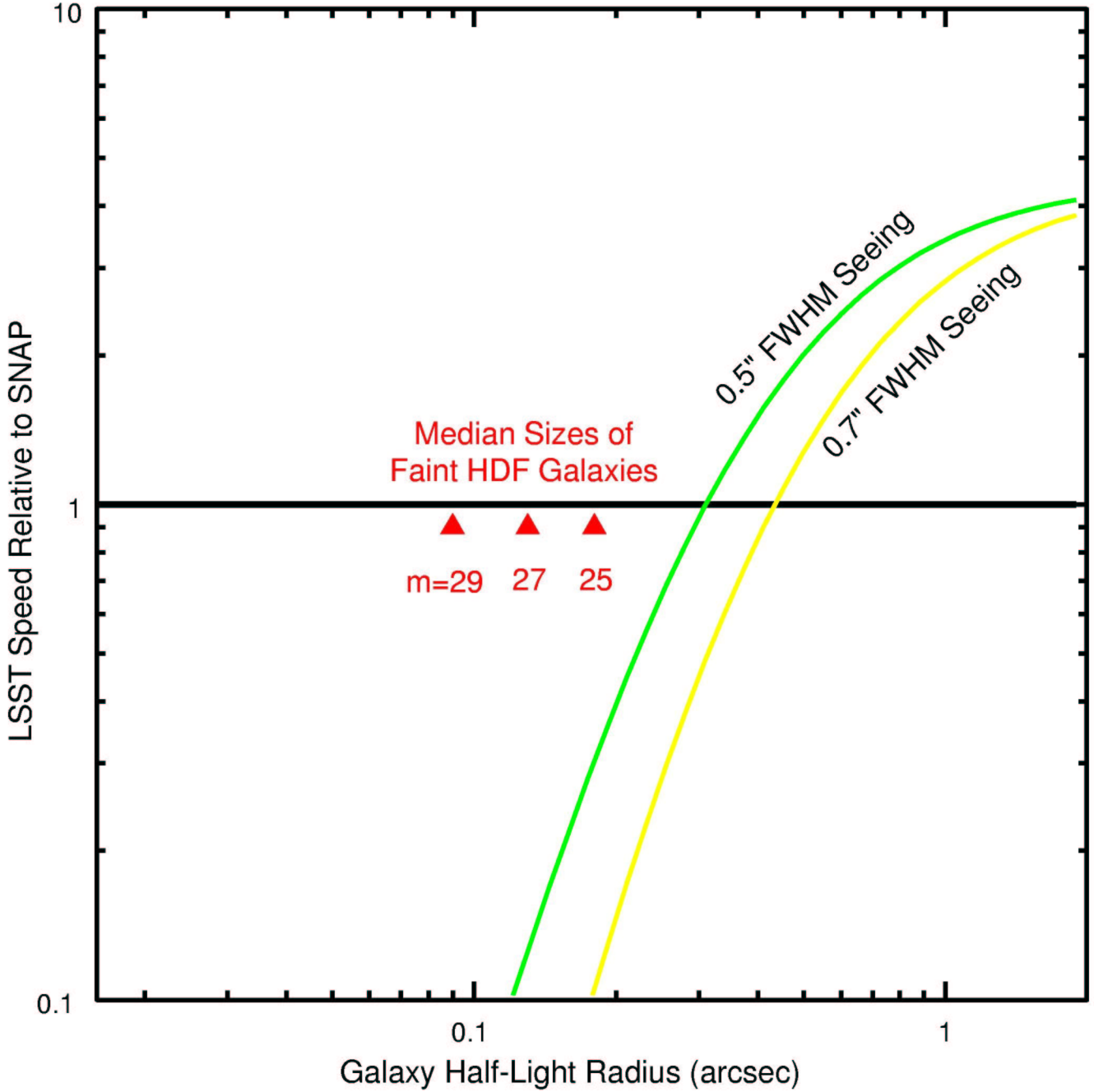
- Wide field in space - large survey area with exquisite image quality
- Depth of survey - reduced shot noise and mapping resolution
- Many photometric bands - evolution of structure as function of redshift
- Small PSF, Thermal stability, stringent optical requirements, multiple exposures - greatly reduced systematics

Bernstein (2001)

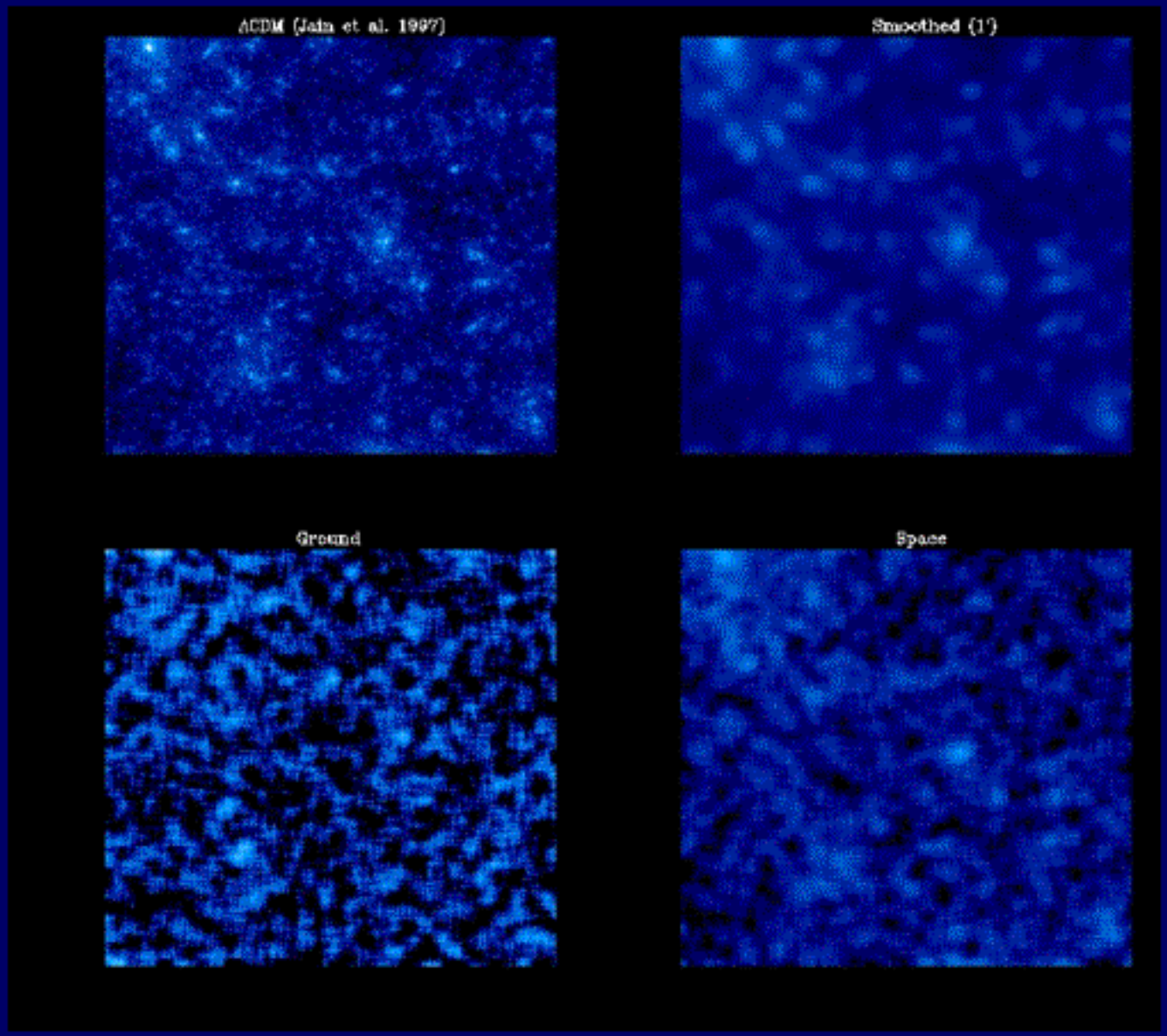


- Brightness and V band wavelength of SNe Ia at peak
- Discovery brightness to prevent Malmquist bias


Radii of co−added images
STIS completeness
80% complete
50% complete
20% complete
STIS psf
Half−Light Radius (arcsec)
0.4
0.2
0
24
26
28
30
STIS 50CCD AB magnitude

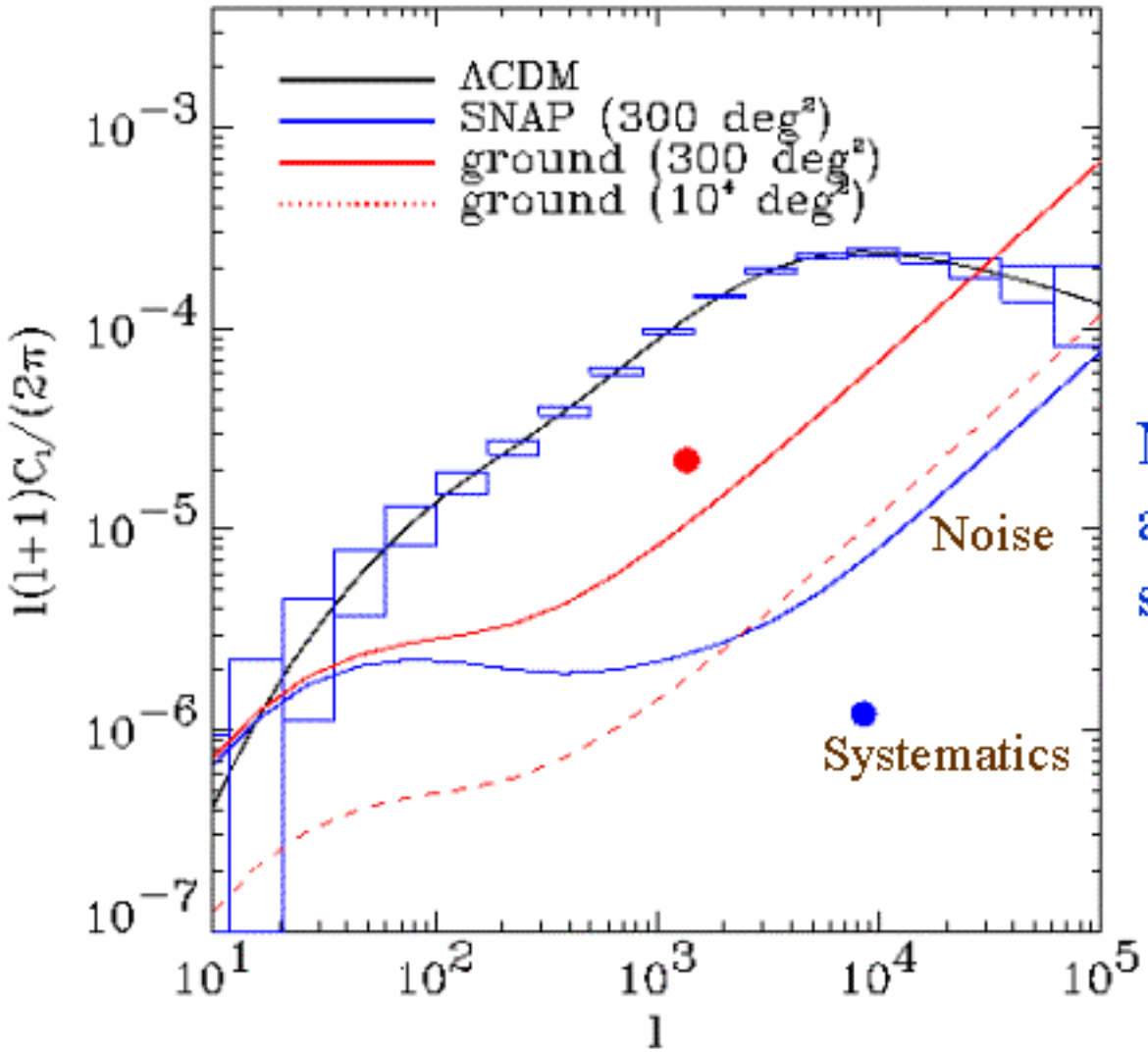

10
1
0.1
0.1
1
LSST Speed Relative to SNAP
Galaxy Half-Light Radius (arcsec)
0.5" FWHM Seeing
0.7" FWHM Seeing
Median Sizes of
Faint HDF Galaxies
m=29
27
25

Mapping the Dark Matter



LCDM
0.5x0.5 deg
Jain et al. 1998

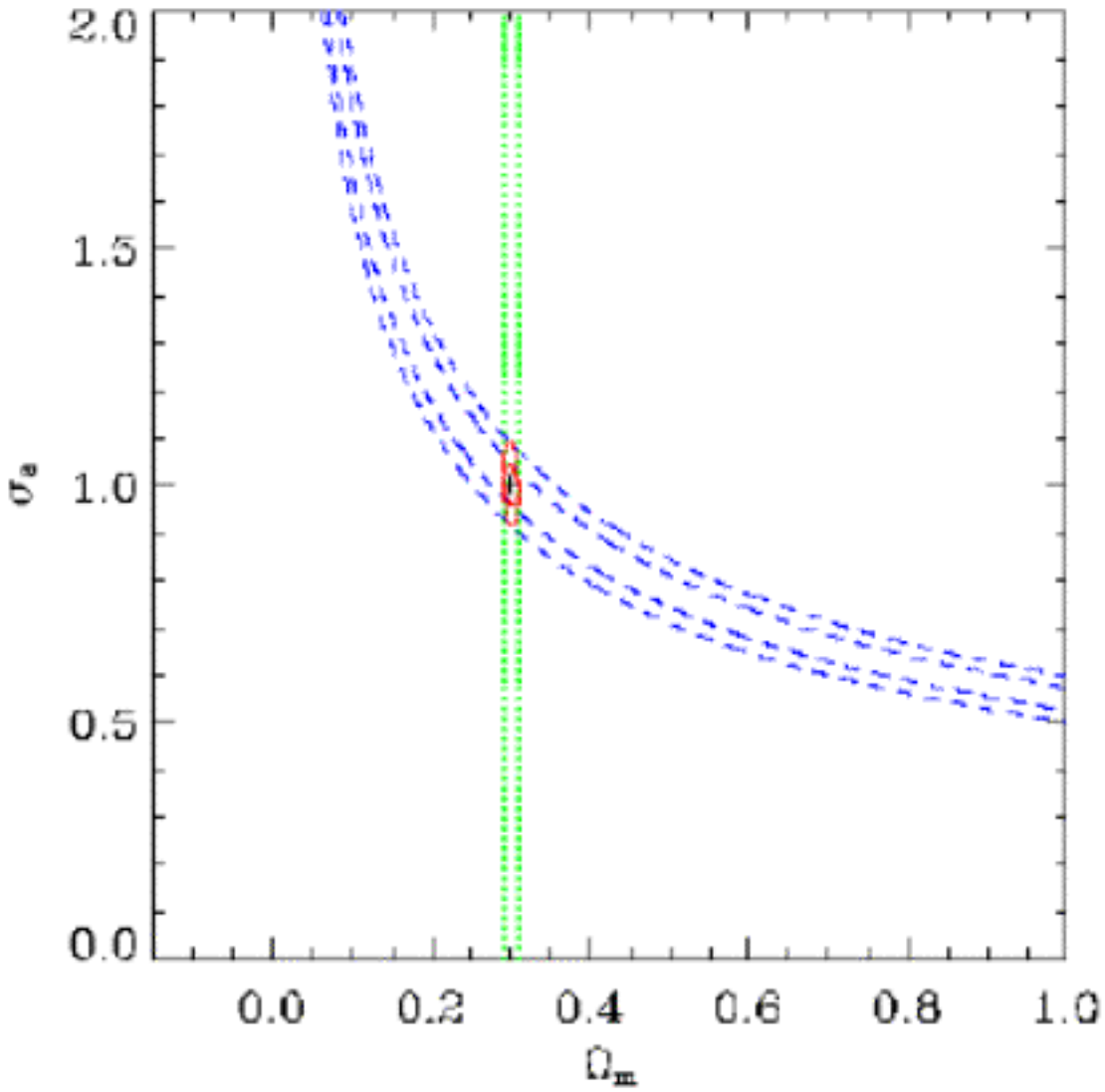
Systematics: Ground vs Space



$$\gamma_{sys} \propto \left(\frac{\theta_{psf}}{\theta_{gal}}\right)^2 \varepsilon_{psf}$$

Noise and Systematics are greatly reduced in space

New cosmological constraints



Cf. Bernardeau et al. 1997

Variance: $\langle \kappa^2 \rangle$

Skewness: $\langle \kappa^3 \rangle$

Data will break current degeneracies (e.g. Ω_M and σ_8; Ω_M and w)

Counting Clusters of Galaxies

Sunyaev Zel'dovich effect

X-ray emission from cluster gas

Weak Lensing

Volume element

$$\frac{dN}{dz\,d\Omega}(z) = \left[\frac{dV}{dz\,d\Omega}(z) \int_{M_{\min}(z)}^{\infty} dM\, \frac{dn}{dM}\right]$$

Simulations:

$$\frac{dn}{dM}(z, M) = 0.315 \frac{\rho_0}{M} \frac{1}{\sigma_M} \frac{d\sigma_M}{dM} \exp\left[-|0.61 - \log(D_z \sigma_M)|^{3.8}\right]$$

growth factor

3: Importance of the Survey Detection Limit

Cluster redshift distribution dN(z)/dz/dΩ

$$\frac{dN(z)}{dz d\Omega} = \frac{c}{H(z)} d_A^2 (1+z)^2 \int_{m_{\rm lim}}^{\infty} dM \frac{dn(M,z)}{dM}$$

Minimum mass of detectable cluster

Limiting mass $M_{lim}(z)$

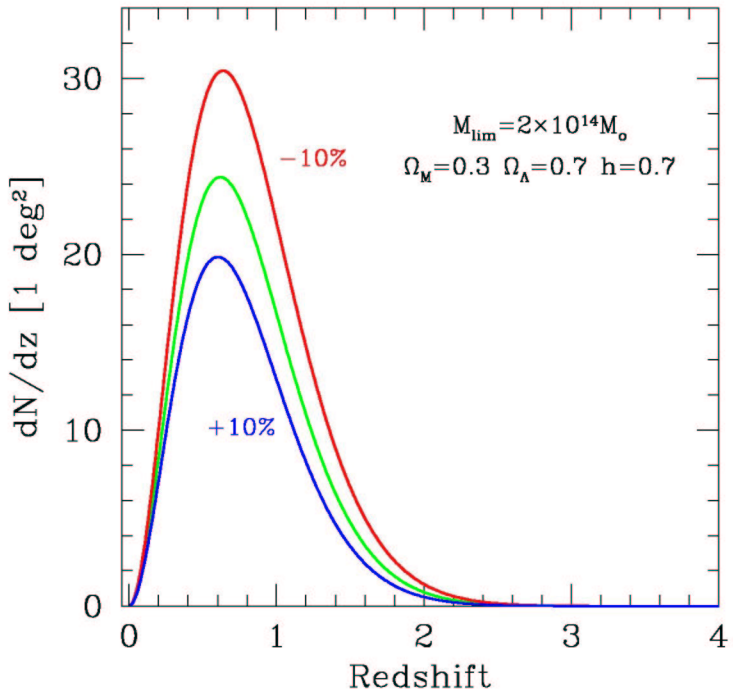
» Connecting cluster virial mass to observables is critically important

- X-ray luminosity or emission weighted temperature
- SZE luminosity
- Weak lensing shear amplitude
- Galaxy light / dynamical estimators

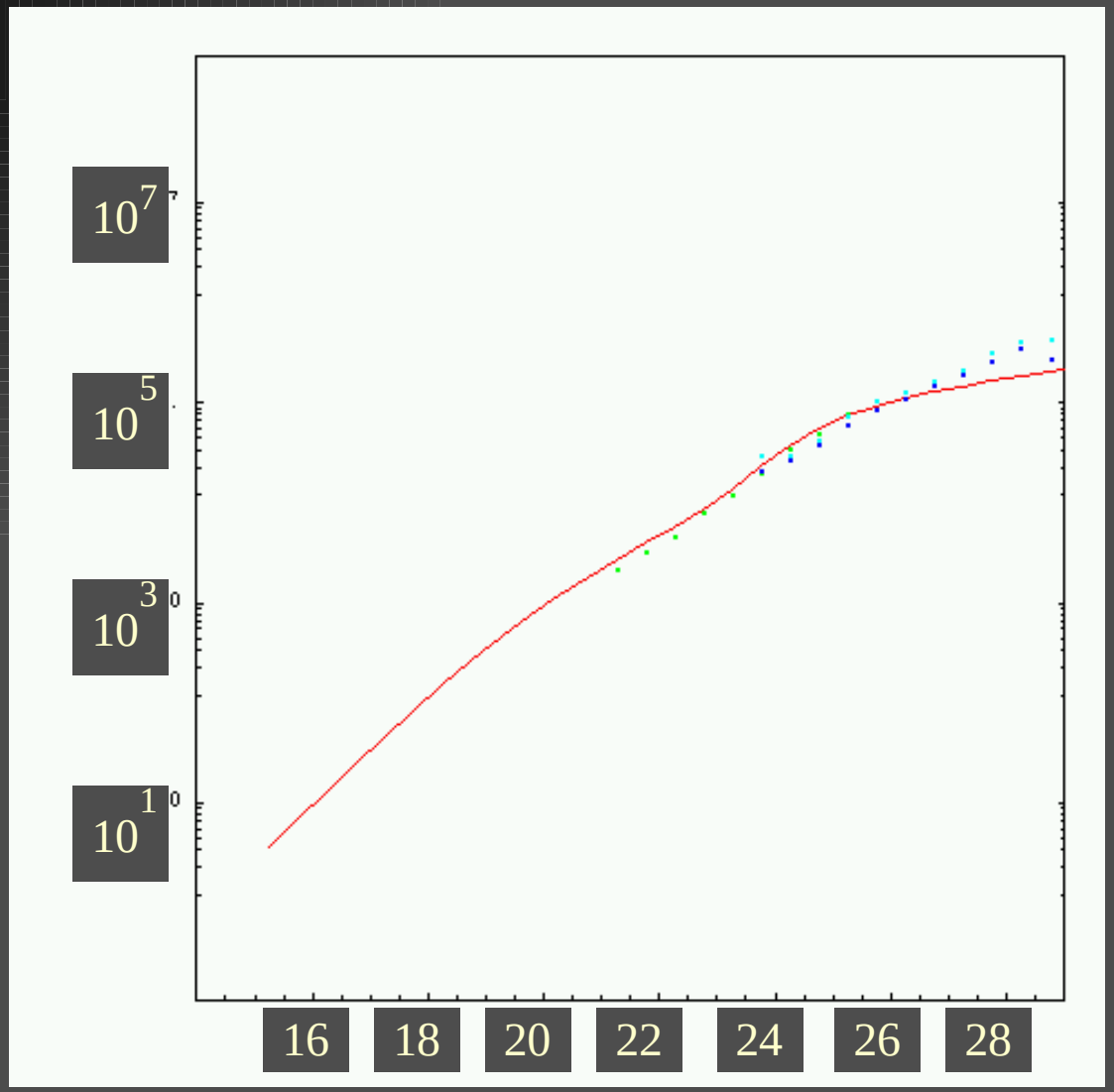
Total SZE flux from Cluster

$$S_{tot} = \frac{-4k\nu^2 \sigma_T T_{cmb}}{m_e c^4} \frac{1}{d_A^2} \frac{f_{ICM} M_{vir} \langle T_e \rangle_n}{\mu_e m_p}$$

Limiting Mass Sensitivity



Weak Lensing:Number Cts of Background Galaxies



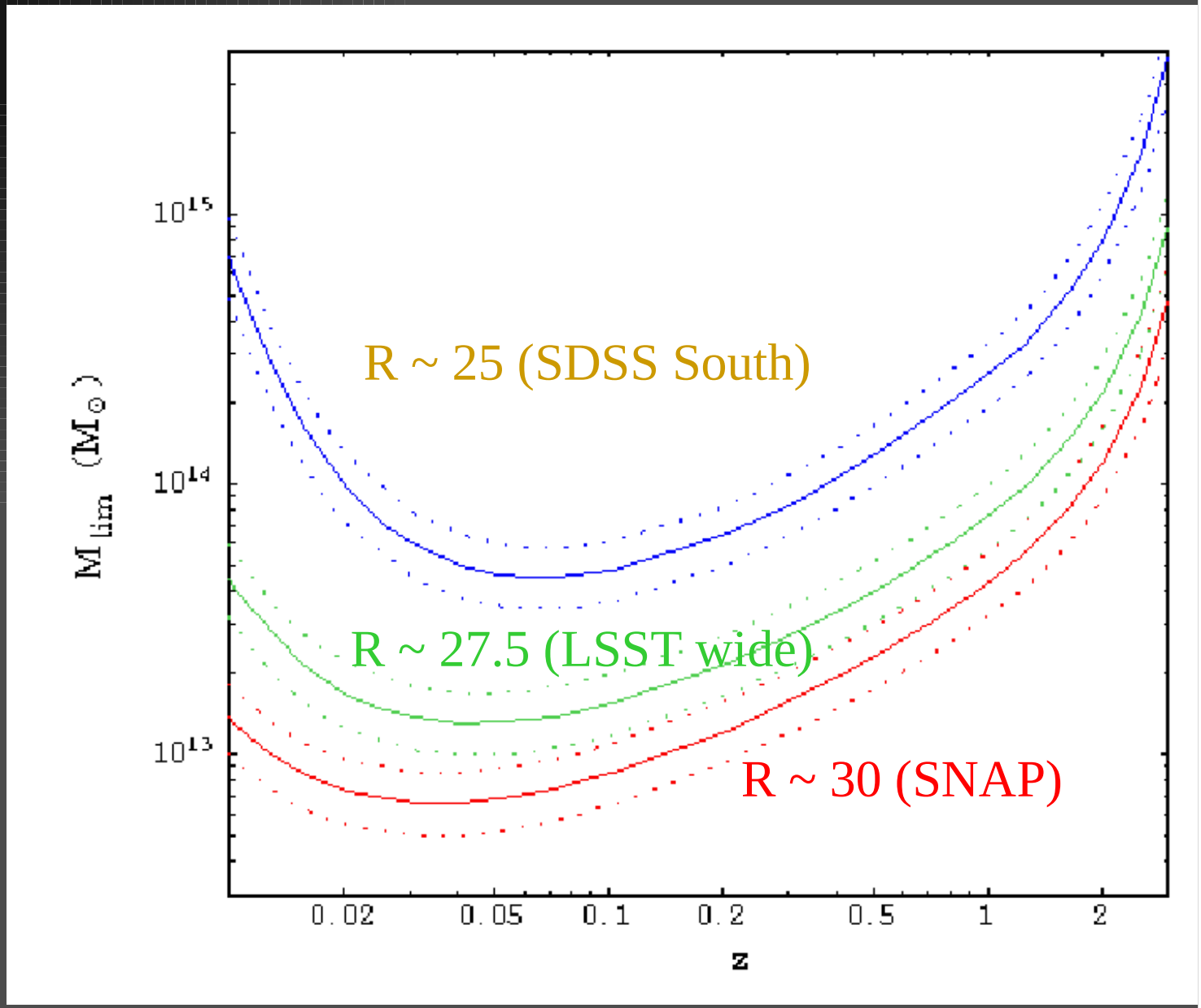
Points: HDF
Curve: extrapolation from SDSS Luminosity Function w/o mergers

Weak Lensing:

Optical Multi-band Surveys of Varying Depth

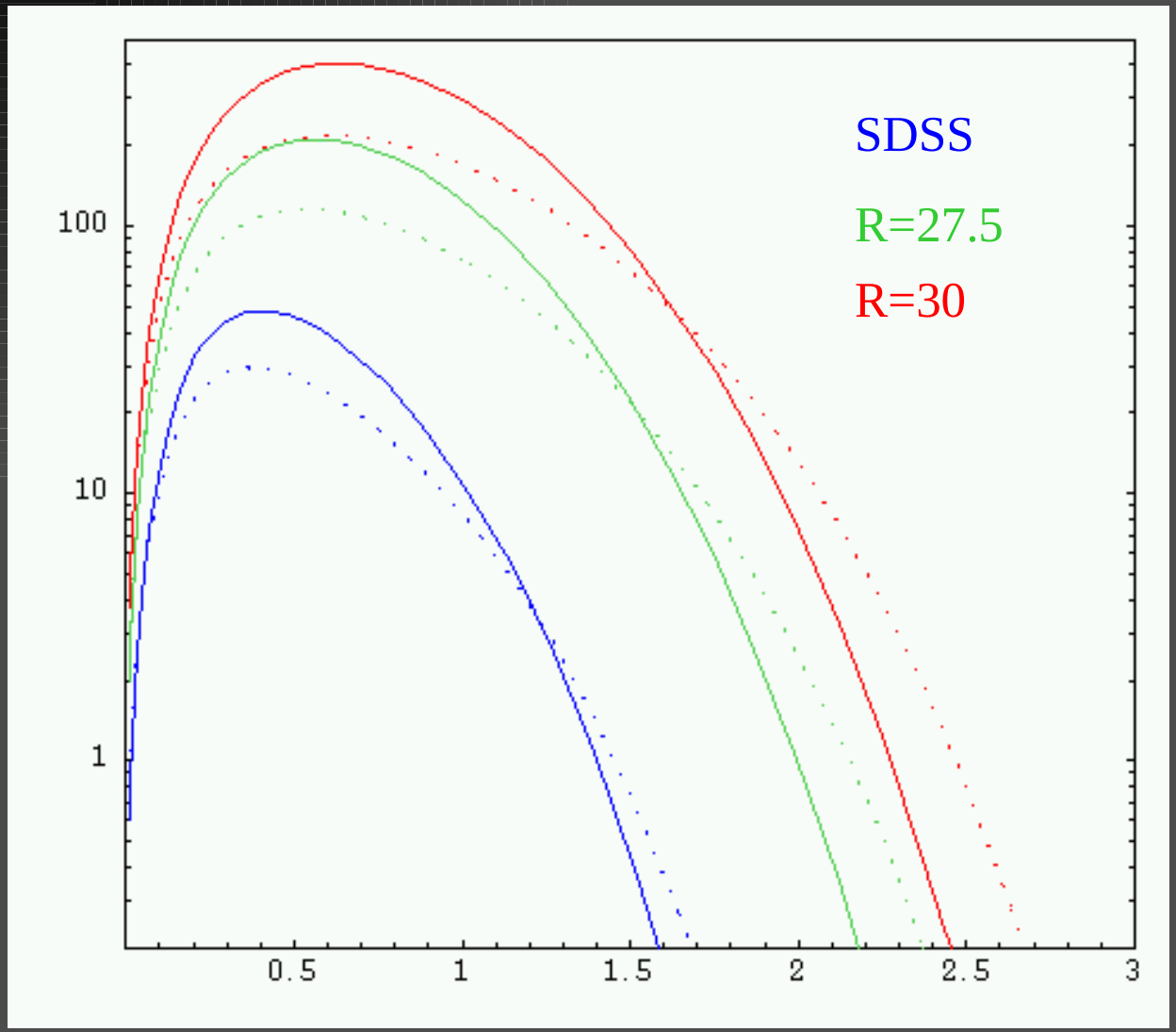
S/N > 5 for Aperture Mass detection

NFW halo profiles



Joffre, Frieman

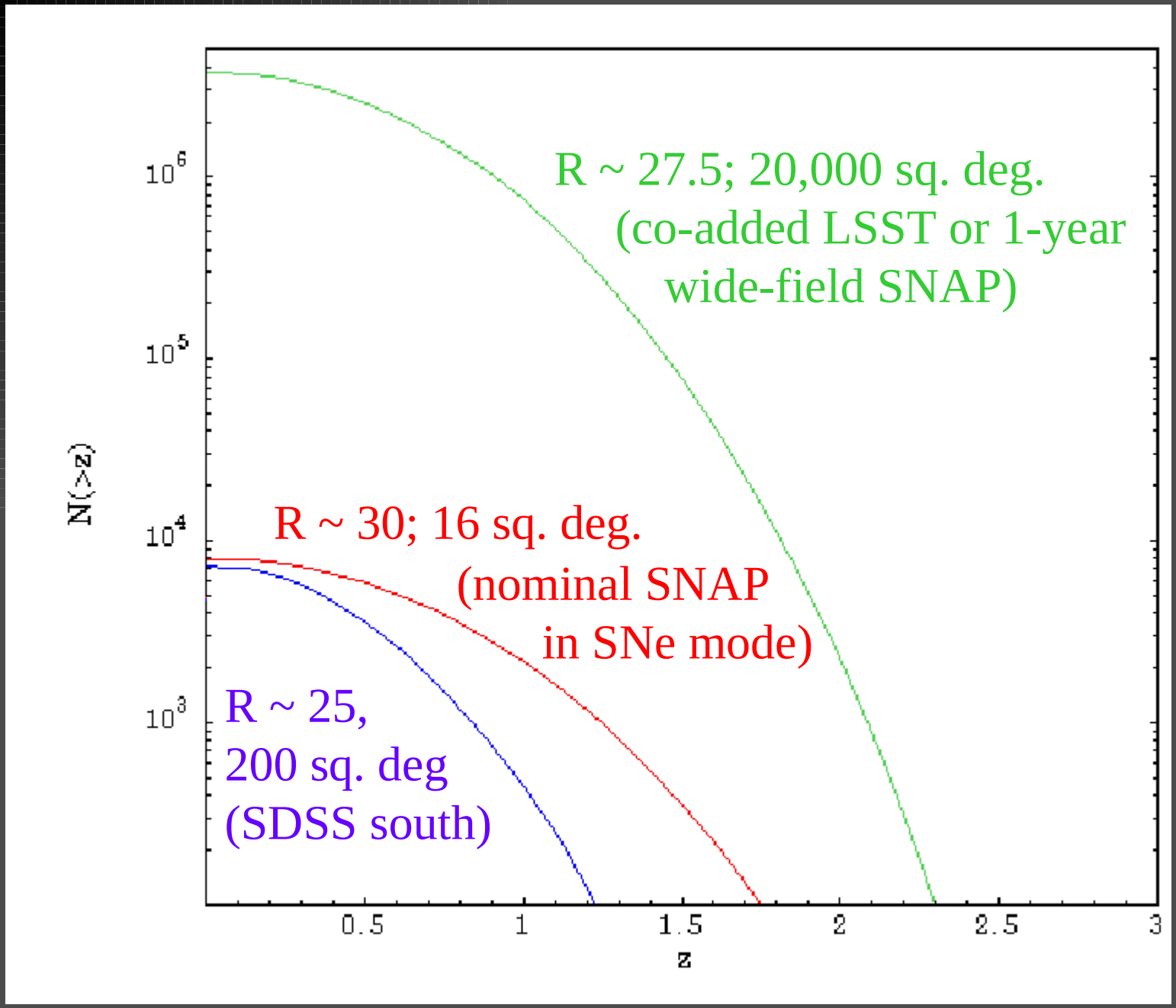
WL Detected Clusters per sq.deg.



dn/dz per sq. deg.

z

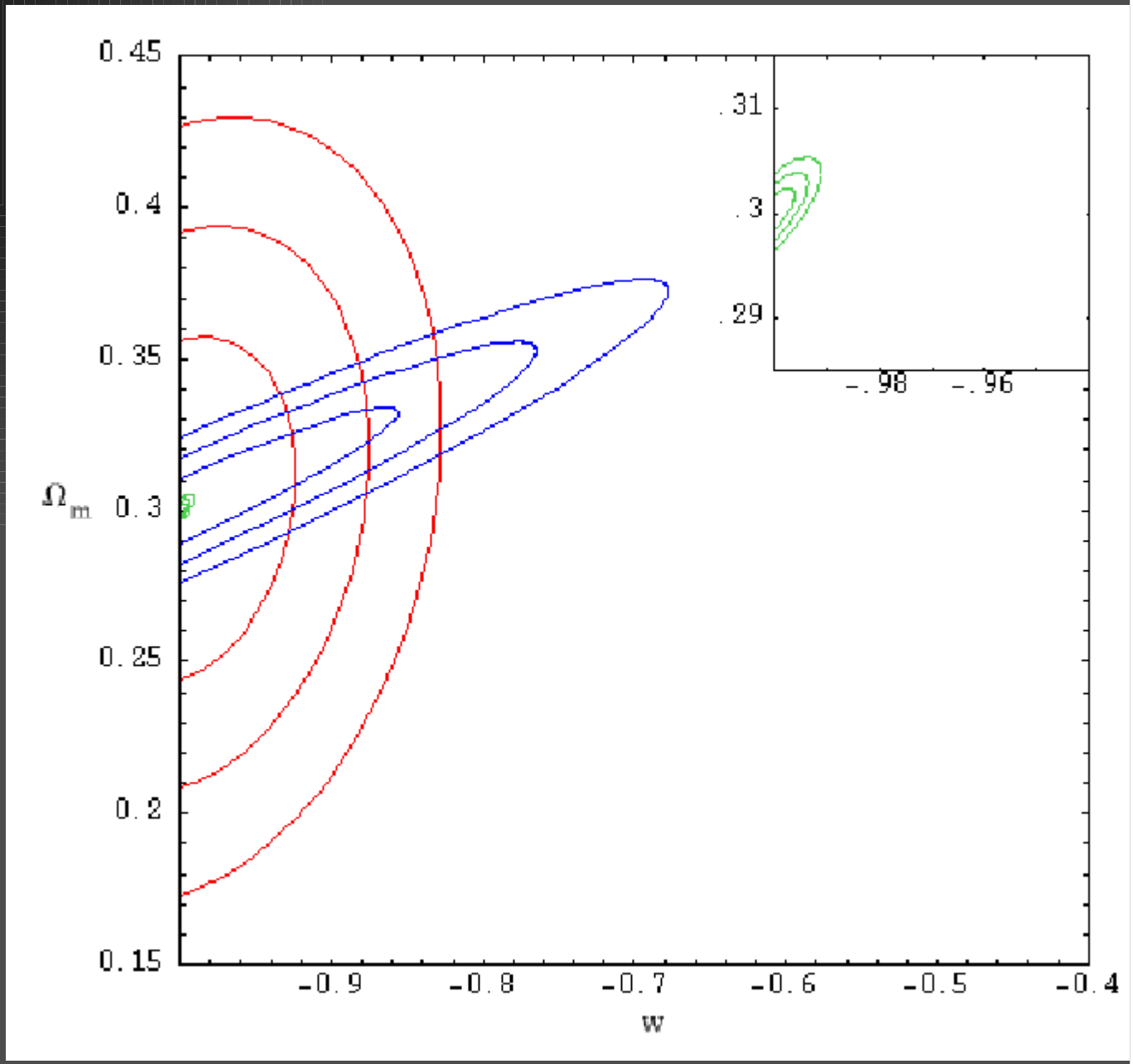
Number of Clusters detected above Threshold



for $\sigma_8 = 1$

Cluster Weak Lensing:

(account for:
projection effects
fuzziness in
Mass limit,
also
variations in
NFW concentration
parameter)



Note: there is more information to be used than N(z)