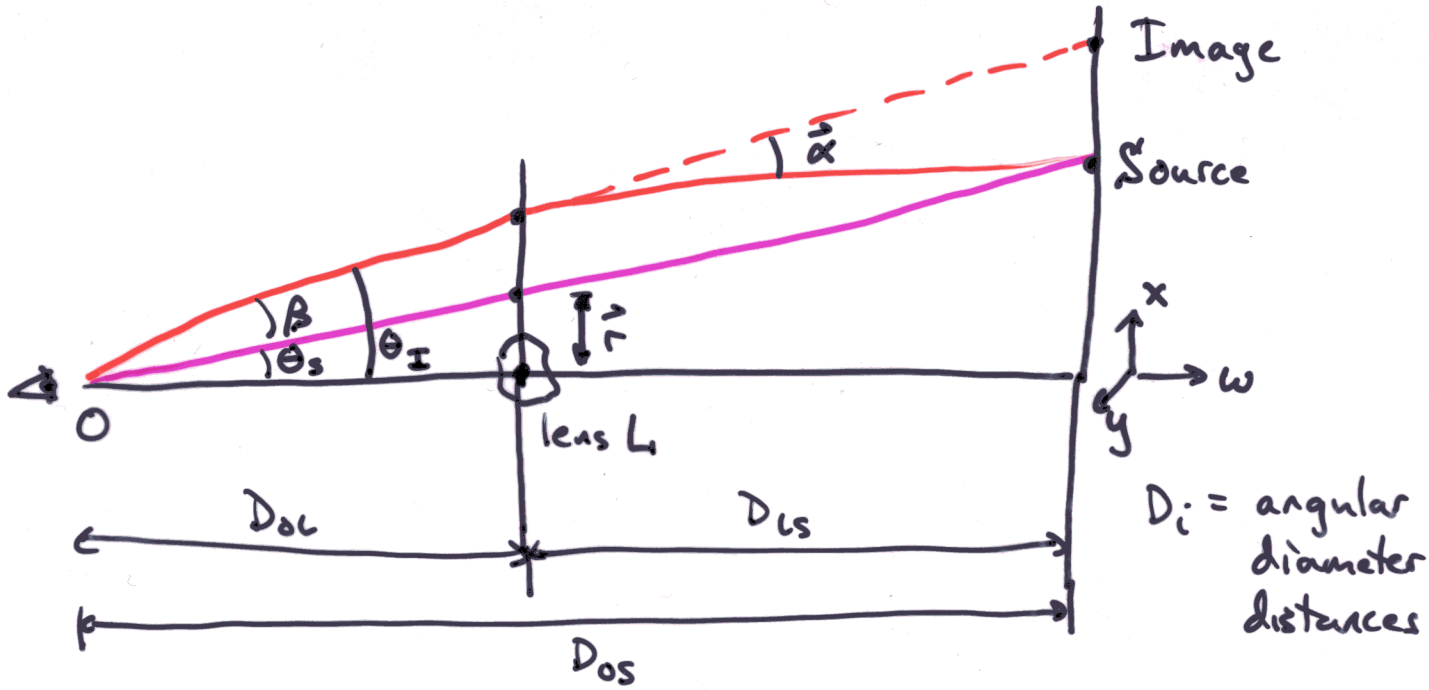
Gravitational Lensing : the Thin Lens Approximation



Lens Equation : $\vec{\theta}_I - \vec{\theta}_s = \vec{\beta}(\vec{\theta}_I) = \dfrac{D_{LS}}{D_{OS}} \vec{\alpha}(\vec{\theta}_s)$

Geodesic Eqn: $\dfrac{d\hat{n}_\gamma}{d\omega} = -2\vec{\nabla}_\perp \Phi$, $\nabla^2 \Phi = 4\pi G\rho$ Poisson

$\Rightarrow$ **Deflection Angle** : $\vec{\beta}(\vec{\theta}) = -2\displaystyle\int_0^{\omega_s} d\omega \left(\frac{\omega_s - \omega}{\omega_s}\right) \vec{\nabla}_\perp \Phi(\vec{\omega}, t)$

$$\equiv \vec{\nabla}_2 \psi(\vec{\theta}_I)$$

$$\boxed{\vec{\nabla}_2 = \left(\frac{\partial}{\partial\theta_x}, \frac{\partial}{\partial\theta_y}\right)}$$

Projected grav. (2-D) potential : $\psi(\vec{\theta}) \equiv \dfrac{2 D_{LS}}{D_{OS} D_{OL}} \displaystyle\int \Phi(\vec{\theta}, \omega) d\omega$

satisfies 2-DIM Poisson eqn : $\nabla_2^2 \psi = 2\dfrac{\Sigma(\vec{\theta})}{\Sigma_{crit}} \equiv 2\kappa(\vec{\theta})$

surface mass density $\int \rho d\omega$

Critical Surface Density for Multiple Imaging: Σ_{crit}

- Minimal condition for multiple images:

  "on-axis" source yields "off-axis" images

  $$\theta_S = 0$$

  Lens Equation: $\theta_I(r) = \dfrac{r}{D_{OL}} = \dfrac{D_{LS}}{D_{OS}}\,\alpha(r)$

  Cylindrically symmetric lens: $\alpha(r) = \dfrac{4GM(r)}{rc^2}$

- Bend angle must be large enough to satisfy lens equ:

$$\frac{4GM(r)}{r^2c^2} \geq \frac{D_{OS}}{D_{OL}\,D_{LS}}$$

- Projected surface mass interior to r =

  Mean surface density " " "

$$\bar{\Sigma}(r) \equiv \frac{M(r)}{\pi r^2}$$

- For multiple images:

$$\bar{\Sigma}(r) \geq \Sigma_{crit} \equiv \frac{c^2 D_{OS}}{4\pi G\, D_{OL}\, D_{LS}}$$

Lensing an Image:

Image surface brightness: $I'(\theta_i) = I\left[M_{ij}\theta_j\right]$

$$= I\left[(\delta_{ij} - \Psi_{ij})\theta_j\right]$$

Weak Lensing: Taylor Expand

$$I'(\vec{\theta}) = (1 + \kappa\hat{K} + \gamma_j \hat{S}_j)\, I(\vec{\theta})$$

$\hat{K} = -\theta_i \partial/\partial\theta_i$ convergence generator

$\hat{S}_1 = -(\theta_1 \partial/\partial\theta_1 - \theta_2 \partial/\partial\theta_2)$ shear generators

$\hat{S}_2 = -(\theta_1 \partial/\partial\theta_2 - \theta_2 \partial/\partial\theta_1)$

Point Spread Function:

Observed image = "true" image convolved with PSF

$$\tilde{I}(\vec{\theta}) = \int d^2\theta \; I(\vec{\theta}-\vec{\theta}')\, P(\vec{\theta}')$$

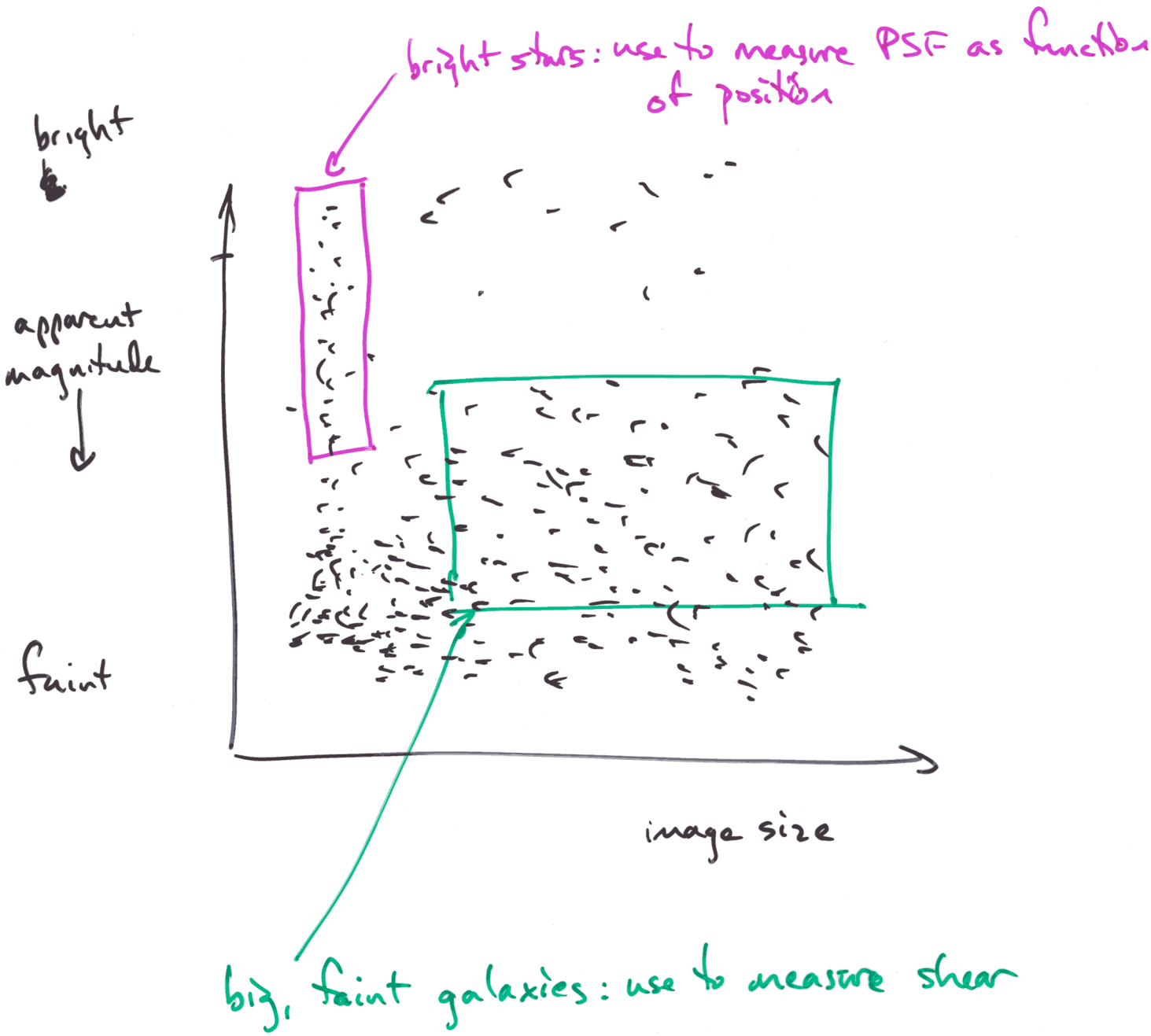
PSF

The smeared image is gravitationally sheared. Need to correct for this:

ideally, deconvolve the PSF.

P: - measure w/ stars

- due to: - atmospheric turbulence + refraction
  - telescope diffraction + optical imperfections
  - ~~atmos~~
  - guiding errors, etc.


bright stars: use to measure PSF as function of position
bright
apparent magnitude
faint
image size
big, faint galaxies: use to measure shear

$$\langle \kappa^2 \rangle = \langle \gamma^2 \rangle =$$

$$\frac{9\pi}{4} \left(\Omega_m H_0^2 \right)^2 \int dR \left(\frac{g(R)}{a(R)} \right)^2 \int \frac{dk}{k^2} k^3 P_\delta(k) W_2^2(kR\theta)$$

$$= \int d\ell \, \ell \, P_\kappa(\ell)$$

$$\Delta P_\kappa(\ell) = \sqrt{\frac{2}{(2\ell+1) f_{sky}}} \left(P_\kappa(\ell) + \frac{\langle e^2 \rangle}{\bar{n}} \right)$$